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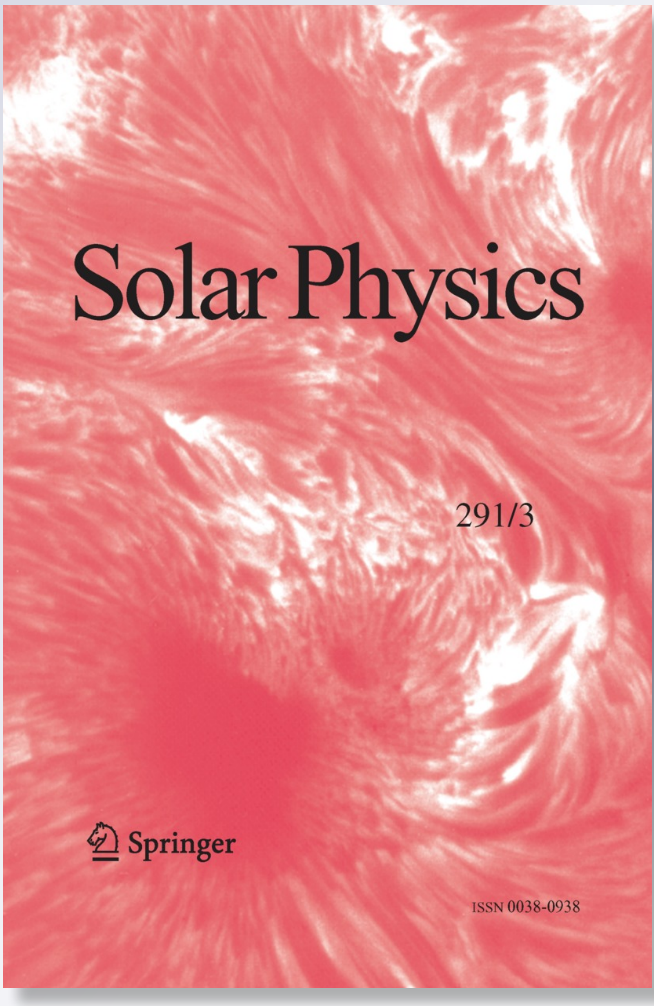

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Possible Estimation of the Solar Cycle Characteristic Parameters by the 10.7 cm Solar Radio Flux

George Lampropoulos[1] · Helen Mavromichalaki[1] · Vasilis Tritakis[2]

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Abstract Two independent methods for estimating basic parameters of the solar cycle are presented. The first of them, the ascending-descending triangle method, is based on a previous work by Tritakis (*Astrophys. Space Sci.* **82**, 463, 1982), which described how the fundamental parameters of a certain solar cycle could be predicted from the shape of the previous one. The relation between the two cycles before and after a specific 11-year solar cycle is tighter than between the two cycles belonging to the same 22-year solar cycle (even-odd cycle). The second is the MinimaxX method, which uses a significant relation in the international sunspot number between the maximum value of a solar cycle and its value 2.5 or 3 years (depending on the enumeration of the even or odd cycle) before the preceding minimum. The tests applied to Cycles 12 to 24 indicate that both methods can estimate the peak of the 11-year solar radio flux at a high confidence level. The data used in this study are the 10.7 cm solar radio flux since 1947, which have been extrapolated back to 1848 from the strong correlation between the monthly international sunspot numbers and the adjusted values of the 10.7 cm radio flux.

Keywords Solar cycles · Solar radio flux · Space weather

1. Introduction

Solar activity is the primary source of quasi-stationary and transient energetic particles as well as electromagnetic emissions, which disturb the interplanetary medium and trigger

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space weather events. Progress in solar physics during the past decades, although impressive, has not yet succeeded in identifying the basic mechanism of solar activity. It is therefore very difficult to derive a reliable prediction of the solar cycle based on fundamental physics laws alone. On the other hand, new and dynamic branches of solar and space physics researches such as space weather, solar variability, and global change all require accurate short-term and medium-term solar activity predictions.

Under such circumstances, it is reasonable to adopt empirical methods that may predict an approximate profile of a forthcoming solar cycle. During the past century, many methods of this category have been developed and many articles have been published on this subject (Pesnell, 2008, 2012, 2014). The common point of all these methods is the statistical analysis of time series of various solar observations, such as relative sunspot numbers, solar optical or electromagnetic emissions, and an analysis of possible geomagnetic and interplanetary precursors. All these methods are classified into three major categories based on the range of prediction they consider. The short-term category contains methods to predict solar activity for the next few hours, days, or weeks. The medium-term methods offer forecasting for the next 11-year cycle, while the long-term methods try to make predictions for several solar cycles ahead. Methods of the last category are rather impractical and have only a philological interest because systematic solar observations cover a short time span, no longer than 160 years, and do not allow us to carry out long-term forecasting (Conway *et al.*, 1998; Hathaway, Wilson, and Reichmann, 1999; Cameron and Schüssler, 2007; Kane, 2008; Pesnell, 2012)

Short-term prediction methods mainly focus on the continuous recording of the solar disk phenomena and the interplanetary medium, seeking for precursors of solar activity changes. Medium-term methods are based on empirical, statistical characteristics of the previous solar cycles that could be applied to the forthcoming cycles.

If a new prediction method is to be developed or some of the existing methods are to be improved, it is necessary to take some basic rules of the solar activity behavior into consideration that were formulated by previous researchers (Waldmeier, 1939; Gnevyshev and Ohl, 1948; Gleissberg, 1973; Gnevyshev, 1977; Mavromichalaki, Marmatsouri, and Vassilaki, 1988; Mavromichalaki *et al.*, 1997, 2003; Mavromichalaki, Belehaki, and Rafios, 1998). The solar electromagnetic spectrum contains a wide range of radio emissions from millimeters to several tens of meters, coming mainly from the solar upper atmosphere (Lin and Forbes, 2000). The solar 10.7 cm (2800 MHz) radio flux (R_{f}), which is around 10^{-19} $\mathrm{Wm^{-2}\,Hz^{-1}}$, is a key parameter describing the radiation from the quiet-Sun upper atmosphere (Tapping and Charrois, 1994; Lin and Forbes, 2000). The bremsstrahlung radiation from the low corona is the dominant source of R_{f} (Tapping and DeTracey, 1990). In addition, the R_{f} flux correlates very well with variations in the ionosphere (Wu *et al.*, 2004). Finally, the index R_{f} represents great advantages over the sunspot number and other solar indices because it is completely objective and observable under all weather conditions.

In the present article, we attempt to determine the main parameters of the next two solar cycles after the current cycle. For this purpose, monthly data of the 10.7 cm solar radio flux and the international sunspot numbers are used. Two estimation methods of the characteristics of upcoming cycles, the ascending-descending triangle and the MinimaxX method, are proposed, and we obtain very interesting results from them.

2. Data and Methods

The solar 10.7 cm radio flux (R_{f}), expressed in solar flux units (1 sfu = 10^{-22} $\mathrm{W\,m^{-2}\,Hz^{-1}}$), constitutes a continuous time-series since 1947. Daily measurements of R_{f}, free from solar

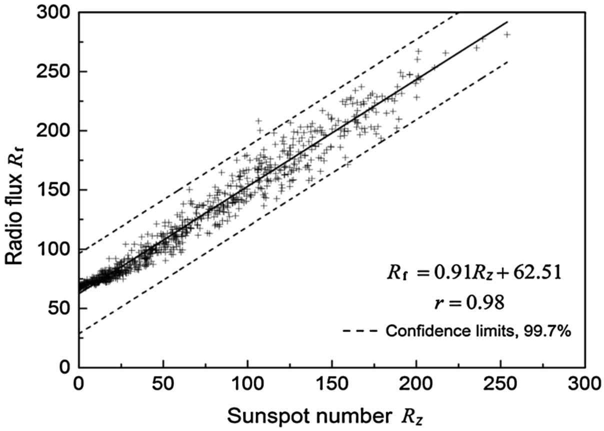

Figure 1 Regression diagram of the monthly means of R_f and R_Z data for the period of 1947 – 2014. Confidence limits $\pm 3\sigma$, where σ is the standard deviation, are drawn on both sides of the regression line.

flare influences, have been provided and tabulated as "observed flux" and "adjusted flux" by the Canadian Solar Radio Monitoring Program (Ottawa). The former are the actual measured values affected by the changing distance between Earth and Sun throughout the year, whereas the latter are scaled to a standard distance of 1 AU. In the following we use the second form (monthly mean of adjusted values), which is more appropriate for our purpose.

On the other hand, reliable time-series of sunspot relative numbers R_Z go back to 1848, when the astronomer Rudolf Wolf started systematic daily sunspot observations in Zurich (Wolf, 1861). On 1 January 1981, the Zurich relative sunspot number program was transferred to the Solar Influences Data Analysis Center (SIDC), Brussels. The final international sunspot numbers, R_i, are evaluated by a method similar to R_Z, but from data collected by a network of twenty-five well-qualified observatories. R_i and R_f data can be easily downloaded from ftp://ftp.ngdc.noaa.gov/STP/space-weather/solar-data/solar-indices/sunspot-numbers/international/listings/listing_international-sunspot-numbers_monthly.txt.

Xanthakis and Poulakos (1985) studied the correlation between solar radio flux R_f and Zurich sunspot numbers R_Z within a very short time range (1957 – 1976) and obtained a correlation coefficient of 0.90. Later, Hathaway, Wilson, and Reichmann (2002), repeating the same study, but for a longer time span (1947 – 2002), indicated a more significant correlation of 0.98. In the present work, we have estimated the correlation coefficient between R_Z and R_f for both the longest time period (1947 – 2014) where real observations are available and the very long period 1848 – 2014 where reliable R_Z observations exist. The close relation between R_Z and R_f supports our effort to continue the data to 1848 – 1947.

Correlations between R_Z and R_f in the period 1947 – 2014 have been calculated by the Pearson product-moment correlation coefficient (r) and Spearman's rank-order correlation coefficient (ρ). The former is the covariance of the two variables divided by the product of their standard deviations, while the latter is a nonparametric measure of statistical dependence between two variables and an estimation of how well the relation between the two variables can be described by a monotonic function. The application of the two methods above has resulted in $r = 0.98$ and $\rho = 0.99$. Since $r^2 = 0.96$, the correlation is significant with a rejection level of 0.01.

The high correlation between R_Z and R_f encouraged us to apply a simple linear regression to estimate the values of R_f back to 1848 – 1947. Figure 1 represents the very tight relation between R_Z and R_f for the period of 1947 – 2014. Figure 2 shows a total time-series of R_f back to 1848 that consists of real (1947 – 2014) and calculated values (1848 – 1946). The

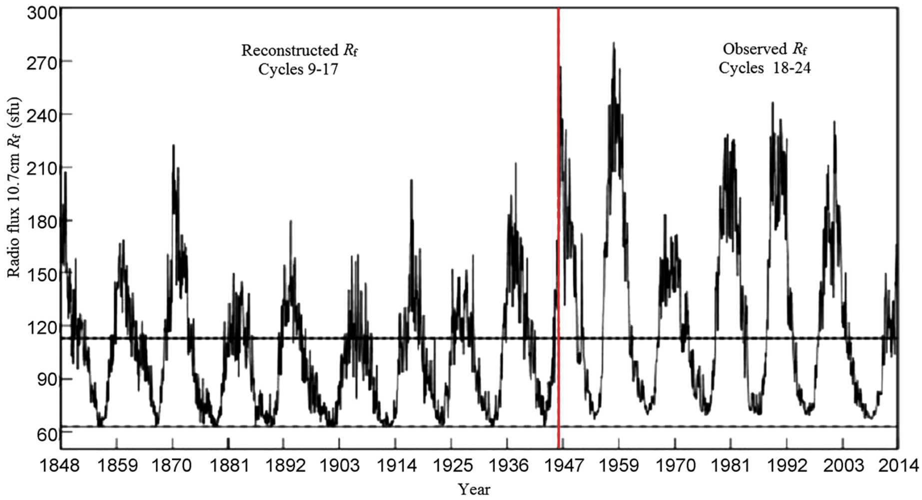

Figure 2 Time series of monthly mean values of solar 10.7 cm radio flux R_f over the period of 1848 – 2013. The reconstructed values for the period 1848 – 1947 are presented together with the observed values for the period 1947 – 2013. The horizontal dashed black lines indicate the minimum amplitude (lower line) and the total mean of R_f (upper line). The vertical red line divides the diagram into the left (reconstructed) and right (observed) areas.

calculated values of R_f could be regarded as a supplementary data set that would strengthen the results obtained from the original R_f alone.

From Figure 2, it is clear that the observed and calculated values are correlated very well, except for two exceptions in 1947 and 2002, which may indicate an intrinsic inaccuracy in our reconstruction. To proceed and estimate characteristic parameters of the forthcoming solar cycles, we analyzed the data mentioned above by two methods, the "ascending-descending triangle" and the "MinimaxX". In the former, fundamental parameters of an 11-year cycle, such as the ascending time and the maximum amplitude, are estimated considering the relation between two successive 11-year cycles that form a 22-year cycle or a pair of cycles in two successive 22-year cycles (Tritakis, 1982). In the latter, we search for a stronger relation between the characteristic values of a solar cycle at a specific time and the peak amplitude of the next cycle. In our present study, we use the Gnevyshev–Ohl (G–O) rule (Gnevyshev and Ohl, 1948), which claims that in pairs of even- and odd-numbered cycles according to Wolf's enumerating system (Wolf, 1881), the odd cycle tends to be more active than the even one. The first method contains statistical and empirical-physical procedures, while the second belongs to the general category of precursor models.

2.1. The Ascending and Descending Triangle Method

A solar cycle can be characterized by two quasi-triangles, called the ascending and descending triangles, which are defined by the ascending and descending branches of the solar cycle and the vertical line drawn from the maximum to the time axis (Tritakis, Mavromichalaki, and Giouvanellis, 2006). The important parameters of these triangles are the slopes of the ascending and descending branches M_a and M_d, the rise time (ascending time) T_a, and the decay time T_d. The regression lines that pass through the (R_f, t) values of the ascending and descending branches define the slopes M_a and M_d of a certain 11-year solar cycle, where t

Table 1 Statistical parameters of Cycles 9 – 24.

Parameter	T_a	T_d	M_a	M_d	R_{fmin}	R_{fmax}	$R_{f,mean}$
Even cycles							
10	4.2	7.0	2.15 ± 0.29	-0.96 ± 0.07	62.6	168.7	102.2
12	4.6	6.1	1.22 ± 0.09	-0.99 ± 0.05	62.5	145.8	95.3
14	5	6.9	1.18 ± 0.11	-0.89 ± 0.09	62.9	161.0	93.8
16	4.3	5.9	1.44 ± 0.13	-1.16 ± 0.11	63.8	152.1	101.1
18	3.6	6.5	4.32 ± 0.45	-1.83 ± 0.13	66.5	267.1	127.4
20	4	7.6	2.26 ± 0.21	-1.06 ± 0.09	69.8	183.1	113.1
22	3.8	6.2	3.83 ± 0.39	-2.08 ± 0.12	69.9	247.2	134.8
24*	4.3	–	1.33 ± 0.22	–	–	166.1	–
Odd cycles							
9	–	8.3	–	-1.17 ± 0.12	–	207.3	–
11	3.4	8.4	3.21 ± 0.36	-1.36 ± 0.14	62.5	222.7	132.5
13	4.3	8.2	1.93 ± 0.19	-0.84 ± 0.09	63.1	180.1	99.2
15	4.1	5.8	1.87 ± 0.14	-1.37 ± 0.12	62.7	203.1	103.9
17	3.9	6.5	2.74 ± 0.21	-1.48 ± 0.13	65.6	212.9	115.1
19	3.3	6.9	4.89 ± 0.36	-2.58 ± 0.21	69.2	281.2	142.2
21	3.8	6.0	3.33 ± 0.29	-2.34 ± 0.22	69.4	229.1	134.7
23	4	8.2	2.42 ± 0.21	-1.51 ± 0.15	67.0	236.2	122.9

*Since Cycle 24 has not been completed yet, approximated values have been assigned.

Table 2 Main differences between even and odd solar cycles.

Even cycles	Odd cycles
Two maxima	One maximum
Smooth rise	Quick rise
Peak-like shape	Saddle-like shape
Point-type maxima	Mesa-type maximum
Quick recovery time (2 – 3 years)	Prolonged recovery time (6 – 8 years)

means the time in years starting from 1 at the first minimum in our data. The times T_a and T_d express the time intervals from the appearance of preliminary indications of a new solar cycle to the maximum and from the maximum to the next minimum, respectively. Table 1 summarizes all the important parameters of Cycles 9 – 24.

Pairs of even-odd cycles according to Wolf's definition present a significant interdependence, which is manifested by strong correlations in the basic parameters of the pair components (even or odd). Differences in the behavior of solar cycles probably correlate with the polarity of the global solar magnetic field at the maximum of a cycle (Hathaway, Wilson, and Reichmann, 1994). Ohl (1966) proposed that this phenomenon originates from distinct conditions corresponding to the parallel and antiparallel states of the solar global magnetic field with respect to the galactic magnetic field. The main differences between even and odd solar cycles are tabulated in Table 2 (Mavromichalaki *et al.*, 1997).

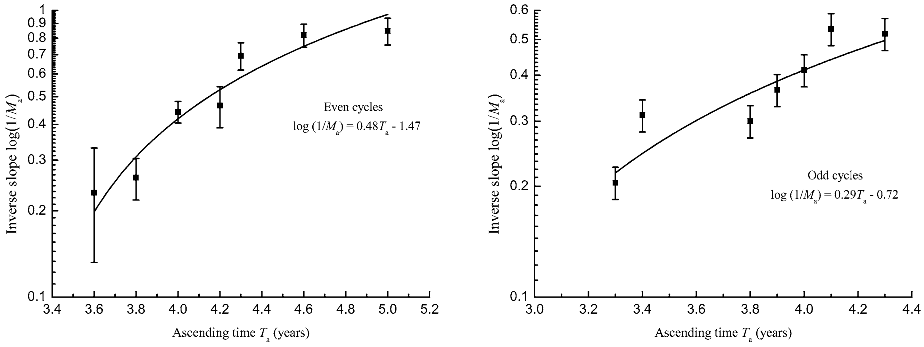

Figure 3 Relation between the inverse ascending slope $1/M_{\mathrm{a}}$ and the ascending time T_{a} for even (left panel) and odd (right panel) cycles.

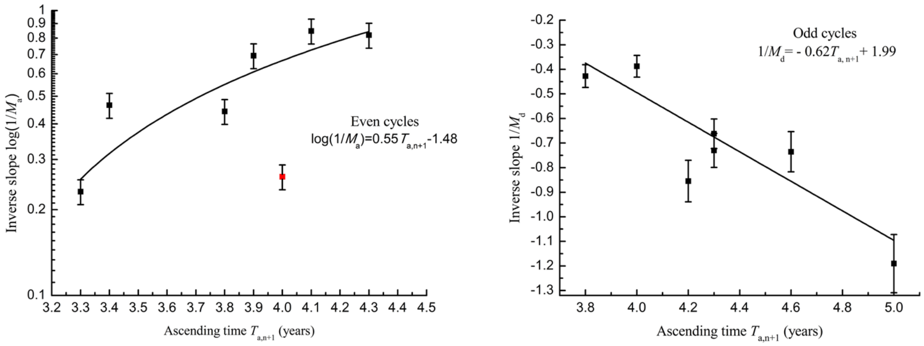

Figure 4 Relation between the inverse ascending slope $1/M_{\mathrm{a}}$ of an even cycle (Cycle n) and the ascending time T_{a} of the next odd cycle (Cycle $n+1$) (left panel). Cycle 22 (red) is excluded because of its abnormally high activity in comparison with its next odd cycle (Cycle 23). The right panel shows the relation between the inverse descending slope $1/M_{\mathrm{d}}$ of an odd cycle (Cycle n) and the ascending time of the next even cycle (Cycle $n+1$).

Tritakis (1982) and Hathaway, Wilson, and Reichmann (1994) have shown the strong correlation between the ascending time and the slope of the ascending branch of both even and odd cycles. This correlation is not present in all the solar activity indices, *i.e.* the sunspot areas (Dikpati, Gilman, and de Toma, 2008). However, in the case of the solar radio flux, the correlation between the ascending time T_{a} and the inverse of the ascending slope is very high, 91 % for even cycles and 90 % for odd cycles (Figure 3).

The relation between the inverse slope of the ascending branch of an even cycle (Cycle n) and the ascending time of the next odd cycle (Cycle $n+1$), as well as between the inverse slope of the descending branch of an odd cycle (Cycle n) and the rising time of the next even cycle (Cycle $n+1$), are depicted in Figure 4. At first glance, the ascending time of the odd cycles is shorter than the ascending time of the even cycles, as expected in the previous work (Mavromichalaki, Marmatsouri, and Vassilaki, 1988). Tight relations between inverse slopes and ascending times of successive cycles give a very good formula for estimating the ascending time of an upcoming cycle by the slope of the previous one, which has completed its ascending branch in the case of even cycles and its descending branch in the case of odd ones. Hence, the slope of the descending branch of the studied odd cycles (Cycles 9, 11,

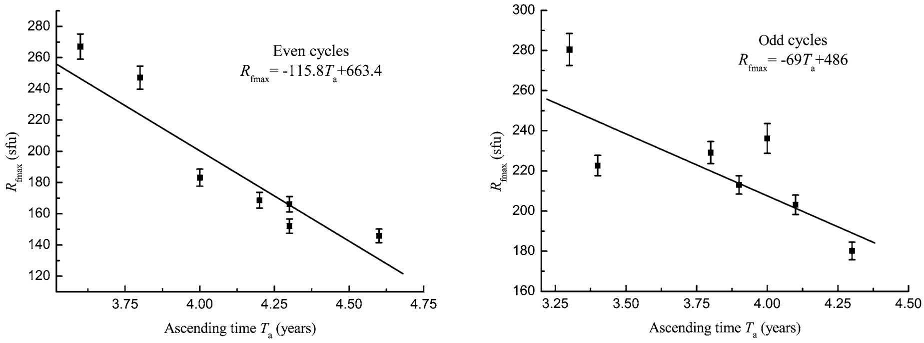

Figure 5 Relation between the maximum amplitude of the solar radio flux $R_{\rm fmax}$ and the ascending time $T_{\rm a}$ for even (left panel) and odd cycles (right panel).

13, 15, 17, 19, 21, and 23) correlates well with the ascending time of the corresponding next even cycles and gives a correlation coefficient of $r = 0.89$. The slope of the ascending branch of the even cycles (Cycles 10, 12, 14, 16, 18, 20, and 22) correlates very well with the ascending time of the next odd cycles ($r = 0.96$). Here Cycle 22 was excluded because of its abnormally high activity, a phenomenon that violates the G–O rule for the pair of Cycles 22 – 23 mentioned in Section 1. The final aspect we focus on is the prediction of the forthcoming solar maximum, which can be derived from a relation between the ascending time and the maximum amplitude of the solar activity (Figure 5). The correlation coefficient obtained is $r = -0.81$ for even and $r = -0.80$ for odd cycles without excluding any cycle with particular behavior.

An estimation procedure for the forthcoming solar maximum starts by calculating for each cycle the ascending ($M_{\rm a}$) and descending ($M_{\rm d}$) slopes, and the ascending ($T_{\rm a}$) and descending ($T_{\rm d}$) times. The times are given in years, while the slopes are given in tenths of a year. Furthermore, in Table 1 the maximum ($R_{\rm fmax}$) and minimum ($R_{\rm fmin}$) monthly mean radio flux values are provided for each cycle, as computed and provided by NGDC. The $R_{\rm fmean}$ characterizes the mean value of radio flux for a whole cycle.

The equations that describe the regression lines mentioned above, as well as the calculated Pearson's correlation coefficients (r), are summarized as follows:

$$\log\frac{1}{M_{{\rm a},n}} = 0.48T_{{\rm a},n} - 1.47, \quad \text{for } n = \text{even cycles},\ r = 0.91, \tag{1}$$

$$\log\frac{1}{M_{{\rm a},n}} = 0.55T_{{\rm a},n+1} - 1.48, \quad \text{for } n = \text{even cycles},\ r = 0.96, \tag{2}$$

$$R_{{\rm fmax},n} = -83T_{{\rm a},n} + 537, \quad \text{for } n = \text{even cycles},\ r = -0.81, \tag{3}$$

$$\log\frac{1}{M_{{\rm a},n}} = 0.29T_{{\rm a},n} - 0.72, \quad \text{for } n = \text{odd cycles},\ r = 0.90, \tag{4}$$

$$\frac{1}{M_{{\rm d},n}} = -0.62T_{{\rm a},n+1} + 1.99, \quad \text{for } n = \text{odd cycles},\ r = -0.89, \tag{5}$$

$$R_{{\rm fmax},n} = -69T_{{\rm a},n} + 486, \quad \text{for } n = \text{odd cycles},\ r = -0.80. \tag{6}$$

Considering that in some cases even cycles present two or more maxima instead of one, it is very confusing how to define the exact date of maximum. For this reason, we specified the maxima by applying a quadratic polynomial fit between the two minima of a certain cycle. After this, we localized the proper date by setting the first derivative of the function that describes the data fit. The first derivative of the solar activity at the date of the maximum is expected to be zero. However, in some cases the date that corresponds to the maximum solar activity between the two minima of a solar cycle is not zero because even solar cycles appear to have two different peaks of maximum activity with a significant time gap between them. In contrast, odd cycles in most of the cases show one maximum, which makes them more predictable. Cycles 21 and 23 are different from the above rule for odd cycles, therefore they are treated like even cycles. The current Cycle 24 presented a complicated maximum period behavior. Peaks of activity in November 2011 (≈ 150 sfu), July 2012 (≈ 140 sfu), May 2013 (≈ 134 sfu), and February 2014 (166.1 sfu) emerged and they made the whole picture very complex. Nevertheless, statistical studies pointed out that the ascending branch of the current cycle has been completed with the date of maximum in May 2013 (Hathaway and Wilson, 2006; Kane, 2007).

2.2. The MinimaxX Method

In the present section, we attempt estimating the maximum of a solar cycle based on the value of the previous minimum. A major difficulty is that each solar cycle minimum contains elements from both the arising and the previous terminating cycles. To overcome this difficulty, it is necessary to pick up a proper parameter in the minimum period that correlates better with the solar maximum. Several authors have presented estimations of a solar cycle maximum in this context. A high negative correlation ($r = -0.77$) between the maximum amplitude of solar activity for a specific cycle and the time duration between two maxima two cycles ago was reported by Du (2006) and Du and Du (2006). Another survey gives a linear correlation of around 70 % between the minimum solar activity of a cycle and its next maximum (Brown, 1976). If Cycle 19 is discarded from our analysis (Brajsa *et al.*, 2009), the correlation of these two quantities is given by

$$R_{\mathrm{max}} = 67.5 + 6.91 R_{\mathrm{min}}. \tag{7}$$

A better version of the last method was given by Cameron and Schüssler (2007), who by experimenting with the time shift of the solar cycle minimum width, found a very good correlation between the solar activity 2.5 years before the minimum and the maximum activity of a cycle. Equation (8) below expresses the relation between these two parameters,

$$R_{\mathrm{fmax}} = 41.9 + 168 R_{\mathrm{tmin\text{-}2.5}}. \tag{8}$$

The application of this relation led to a correlation coefficient $r = 0.80$, which is the best case among similar methods based upon precursors of the activity magnitude of a coming solar cycle (Petrovay, 2010). It is necessary to highlight that all the works mentioned above treated the sunspot numbers as a measure of the solar activity. In contrast, in our case (MinimaxX), we treat the 10.7 cm radio flux (R_{f}) as a measure of the solar activity, while we introduce the even-odd cycle separation to increase the correlation between solar activity at a certain time before the minimum and the maxima values of these cycles in very high levels. We have already mentioned that a very strong correlation ($r = 0.98$) between the radio flux and the sunspot numbers exists, which is defined by the equation

$$R_{\mathrm{f}} = 0.91 R_{\mathrm{Z}} + 61.74. \tag{9}$$

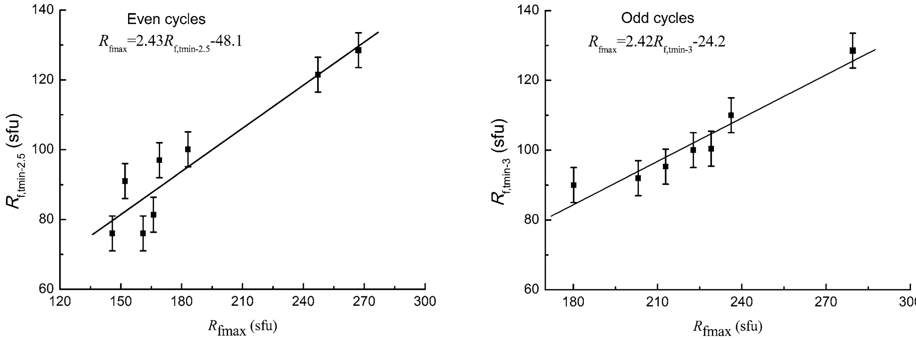

Figure 6 Relation between the solar radio flux (R_f) 2.5 years before the minimum ($R_{f,tmin-2.5}$) and the next maximum R_{fmax} for even cycles (left panel). The right panel shows the solar radio flux 3 years before the minimum ($R_{f,tmin-3}$) *versus* the next maximum R_{fmax} for odd cycles (right panel).

Odd cycles exhibit a more rapid rise to the maximum of the solar activity, while their decay to the next minimum is slower, generally. In contrast, even cycles tend to present a prolonged increase to the maximum and shorter drop to the minimum. Hence, it is reasonable that a precursor model is applied separately in even and odd cycles. An experiment in the minimum period of the solar cycles revealed that the best relation between the minimum-time parameters and the maximum of each cycle is obtained when we take R_f values 30 months before the minimum of even cycles and 36 months before the minimum of odd cycles. Namely, there is a time shift of 6 months between the minima of even and odd cycles. This is probably due to the higher activity of the odd cycles, which triggers a more rapid kick-off of a cycle, which is consistent with the G–O rule (Gnevyshev and Ohl, 1948).

The relations between the maximum and the value 2.5 years before the minimum for even and odd cycles are depicted in Figure 6. Equations (10) and (11) below express the relevant relations (regression fits). After all, if we are certain that a solar cycle has started, we can estimate its forthcoming maximum amplitude with accuracy 90 % and 94 % for even and odd cycles, respectively.

$$R_{fmax} = 2.43 R_{f,tmin\text{-}2.5} - 48.1 \quad \text{for even cycles}, \ r = 0.93, \tag{10}$$

$$R_{fmax} = 2.42 R_{f,tmin\text{-}3} - 24.2 \quad \text{for odd cycles}, \ r = 0.95. \tag{11}$$

The results of these relations applied to Cycles 10 – 24 are shown in Figures 7 and 8. The observed and the values calculated by Equations (2) and (5) of the ascending time of Cycles 10 – 24 are depicted in Figure 7. Figure 8 is similar to Figure 7, but represents the observed and the values calculated by Equation (8) of the maximum radio flux for Cycles 10 – 24. In Figures 7 and 8 a close relation between the observed and calculated values is evident, which is very promising for a successful estimation of the same elements in the forthcoming cycles. However, a significant discrepancy in the values of the ascending time of Cycle 23 and the maximum R_f value of Cycle 14 is evident.

The application of the MinimaxX model to all the prior solar cycles studied (Cycles 10 – 24), to examine its ability to provide a reliable prediction, revealed a very high accuracy (92 %) between the observed and calculated values of the maximum 10.7 cm radio flux for these solar cycles. The predicted (solid line) and observed values (dotted line) are shown in Figure 9, where a good coincidence between them is obvious.

The present Cycle 24 is still in progress, but seems to have already approached its maximum in February 2014 and now is in its descending phase to the minimum. If this is so, the

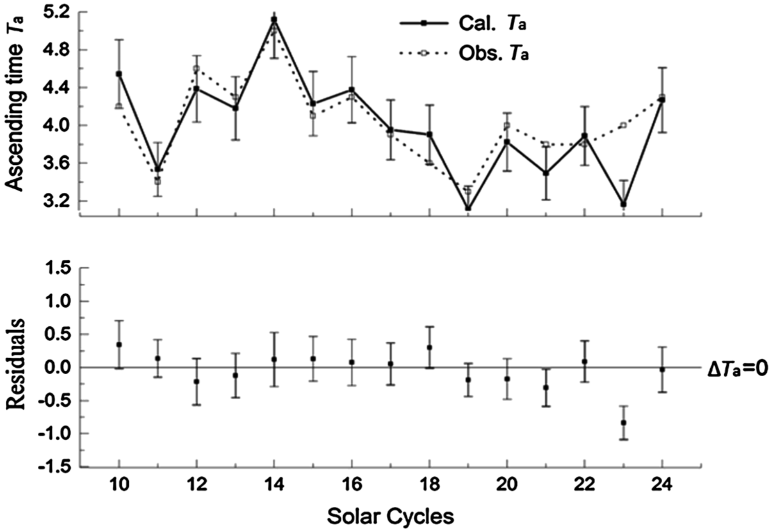

Figure 7 Calculated (solid line) and observed (dashed line) values of the ascending time T_{a} for Cycles 10 – 24 (upper panel). The lower panel shows the difference between the calculated and observed values (ΔT_{a}) together with the error bars.

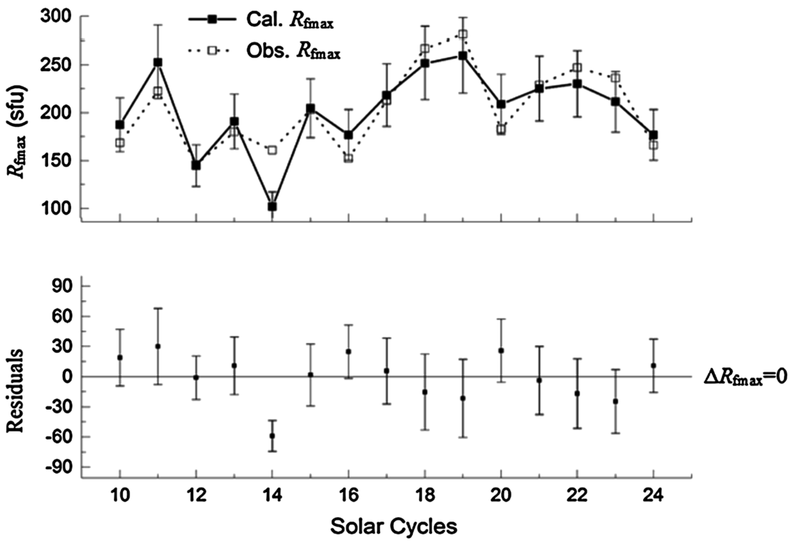

Figure 8 Calculated (solid line) and observed (dashed line) values of the maximum radio flux for Cycles 9 – 24 after application of the asceding-descending method (upper panel). The lower panel shows the difference between these values (ΔR_{f}) together with the error bars.

prediction for the maximum amplitude was successful from both methods we treated in this article. The prediction accuracy approached ≈ 90 %, while the prediction for the ascending time to the maximum, which can be treated only by the first method, reached an accuracy of 100 %.

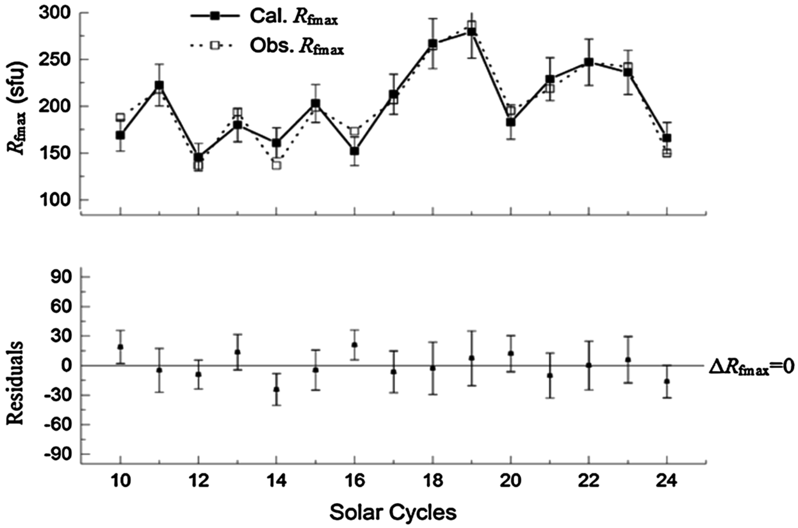

Figure 9 Calculated (solid line) and observed (dashed line) values of the maximum radio flux for Cycles 9 – 24 after applying the MinimaxX method (upper panel). The lower panel shows the difference between the calculated and the observed values (ΔR_f) together with the error bars.

Table 3 Comparison of seven different groups for the prediction of forthcoming Cycle 25.

Parameter	Current method	Hiremath (2008)	Rigozo *et al.* (2011)	Pishkalo (2008)	Yoshida and Sayre (2013)	Helal and Galal (2013)	Du and Du (2006)
T_a (year)	4.1 ± 0.3	–	4.2	4.4 ± 0.7	–	4	–
$R_{f,max}$ (sfu)	204 ± 41	–	–	–	–	–	–
R_{max}	157 ± 33	110 ± 10	132.1	112.3 ± 33.4	108.8 ± 15.1	118.2	111.6 ± 17.4

If the current solar cycle has completed the ascending branch, which is partly hypothetical, we can apply the first method (ascending-descending triangle) to predict the behavior of the next solar cycle (Cycle 25). The MinimaxX model cannot be applied before the start of the next solar cycle. The prediction results for the forthcoming Cycle 25 after applying the ascending-descending triangle method are shown in Table 3, together with the error estimates. In this table a comparison with the predicted results for Cycle 25 according to previous works (Du and Du, 2006; Hiremath, 2008; Pishkalo, 2008; Rigozo *et al.*, 2011; Yoshida and Sayre, 2013) are also presented. The good agreement with our proposed method indicates the importance of this model.

3. Discussion and Conclusions

The interdependence between even-odd solar cycles leads to strong correlations among several parameters of them, such as the ascending time, the slope of ascending-descending branches, and the maximum/minimum values of the cycles. This finally helped us to come to the following conclusions:

1) There is a very strong correlation between the monthly means of the 10.7 cm solar radio flux R_{f} and the monthly means of the sunspot numbers R_{Z}, on the order of 98 – 99 %, depending on the statistical method we apply (Pearson and Spearman correlations). This means that the solar radio flux is a reliable index of solar activity especially for medium-term variations.
2) Although it is too early to come to a certain conclusion, our prediction about the solar maximum of Cycle 24 apparently was successful. There is a high possibility that the date of this maximum occurred in May 2013, while the maximum value of the solar radio flux was 166.1 sfu. In addition, both models give, with very high accuracy, a medium to low activity, which is consistent with the prolonged ascending branch of Cycle 24.
3) Bearing the predicted values of the solar radio flux maximum amplitude and the ascending time of Cycle 24 in mind, we assume that the next solar cycle (Cycle 25) will be more intense, reaching an amplitude of 204 sfu, which is higher than the value of 161 sfu of the current cycle (Cycle 24). This is a preliminary indication of a quasi-periodic oscillation of the solar activity amplitude that may introduce a long-term, 70 to 100-year cycle, known as Gleissberg's cycle (Sonett, Finney, and Berger, 1990; Braun *et al.*, 2005). Furthermore, a similarity in the behavior between Cycles 15 and 25 should be mentioned.
4) Predictions by the ascending-descending triangle method for more than one cycles in advance are possible, but only remain academic since the accuracy in these predictions falls to very low levels (below 65 % for two cycles ahead and below 40 % for three cycles ahead).
5) The MinimaxX model is a very simple but sufficiently accurate predictive model of the maximum activity of the next solar cycle. The idea is to find a relation that links the activity of a solar cycle at a certain time with the maximum activity of the next solar cycle. After several statistical studies, we found that the monthly mean value of solar radio flux that is recorded 30 months before the minimum of an even cycle and 36 months before an odd one is very well correlated with the corresponding value of the next maximum. If we complete this result with the relation between the maximum amplitude and the ascending time from the ascending-descending triangle method, we can predict the time at which the maximum will occur with a significant accuracy. A limitation in this procedure is that the solar minimum must have already been detected, that is, several months should have passed before we are sure of the exact minimum date. Solar cycles with intense activity and short ascending time like odd cycles make the application of this method less effective.
6) The application of statistical models offers a good estimate of the solar activity on satisfactory confidence levels, although there is a serious lack of physical justification. Since all solar parameters seem to behave in a non-linear way, we are planning to introduce non-linear methods in the prediction models in the near future (Petrovay, 2010; Wintoft, 2011).

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