COSMIC-RAY INTENSITY RELATED TO SOLAR AND TERRESTRIAL ACTIVITY INDICES IN SOLAR CYCLE No. 20

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Abstract. The 11-year modulation of cosmic-ray intensity is studied using the data from nine world-wide neutron monitoring stations over the period 1965–1975. From this analysis the following relation among the modulated cosmic-ray intensity I, the relative sunspot number R, the number of proton events N_p and the geomagnetic index A_p has been derived which describes the long-term modulation of cosmic rays

$$I = C - 10^{-3}(KR + 4N_p + 12A_p)\,,$$

where C is a constant which depends on the rigidity of each station, and K is a coefficient related to the diffusion coefficient of cosmic rays and its transition in space. The standard deviation between the observed and calculated values of cosmic-ray intensity is about 5–9%. This relation has been explained by a generalization of the Simpson solar wind model which has been proved by the spherically symmetric diffusion–convection theory.

1. Introduction

The inverse correlation between cosmic-ray intensity and solar activity in the 11-year variation was first pointed out by Forbush (1958) and has been studied in detail by many researchers (see reviews by Rao, 1972; Pomerantz and Duggal, 1974; Moraal, 1976). According to these studies the time lag between cosmic-ray intensity and solar activity varies from several to 12 months, depending on the solar cycle and on the activity index adopted (Balasubrahmanyan, 1969; Dorman *et al*, 1977). Xanthakis (1971) has found a time lag of one year between the cosmic-ray intensity and the solar activity index I_a for the 19th solar cycle. Nagashima and Morishita (1980b) have pointed out that the hysteresis between the solar activity maximum and the cosmic-ray intensity minimum is 9, 1, 10–11 and 2 months for each of the solar cycles 17, 18, 19 and 20, respectively. Other

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indices of solar activity, such as geomagnetic index A_p or coronal green line intensity, appear to reduce the hysteresis effects considerably. Moreover, the time lag depends on solar activity (Wang, 1970) and is shorter in the decreasing phase of activity than in the increasing phase (Simpson, 1963). Also it decreases as the cosmic-ray rigidity increases. Recently the hysteresis mode of the Sun's effect on cosmic-ray flux arriving from the Galaxy to the Earth's orbit has been shown to result from (1) the large size of the modulation region, (2) the variations of the mean sunspot helio-latitude from high to low latitudes throughout the 11-year cycle and (3) the finite time of galactic cosmic-ray diffusion to the modulating region, which is essentially a function of particle energy (Dorman and Soliman, 1979).

Studies of long-term modulations of cosmic rays in interplanetary space give valuable information about the electromagnetic state in the heliomagnetosphere and about the origin of cosmic rays. Thus, a large amount of data concerning the rigidity dependence of the long-term variation of cosmic rays and its relationship with other solar and terrestrial parameters have now been used in comparisons with various theoretical predictions (Rao, 1972).

In a previous work, Xanthakis (1971) has given a quantitative relationship among the cosmic-ray intensity obtained from Mt Washington station's data, the solar activity index I_a and the number of proton flares N_{PF} for the 19th solar cycle. Chirkov and Kuzmin (1979) have shown that the 11-year cosmic-ray intensity I_{PW}, from data of the ionization chamber in Yakutsk for the 19th and 20th solar cycles, can be expressed as

$$I_{PW}(\%) = -0.008\,W - KC_i + A\,, \tag{1}$$

where W is the Wolf number, C_i the geomagnetic index and K and A are constants dependent on the solar cycle. Recently Nagashima and Morishita (1980b) have also used the sunspot number R and the geomagnetic index AA to compute the modulated cosmic-ray intensity.

In this work it is proposed to find a general relationship between the intensity of galactic cosmic rays and the most appropriate solar and terrestrial activity indices which are influenced by the cosmic-ray modulation. For this purpose we have taken account of the following indices: the relative sunspot number R, the number of proton events N_p and the geomagnetic activity index A_p. This relation will be interpretated from a generalization of Simpson's coasting solar wind model (1963).

2. Data Analysis and Results

In order to study the long-term modulation in cycle number 20, data of cosmic-ray intensities have been used from nine neutron monitoring stations

TABLE I
Stations whose data have been utilized in this analysis

Station (Super NM-64)	Height (m)	Geographic latitude (deg)	Coord. longitude (deg)	Threshold rigidity (GV)
Alert	57	82.50N	62.33W	0.00
Thule	260	76.60N	68.80W	0.00
McMurdo	48	77.90S	166.60E	0.01
Inuvik	21	68.35N	133.72W	0.18
Goose Bay	46	53.27N	60.40W	0.52
Deep River	145	46.10N	77.50W	1.02
Kiel	54	54.30N	10.10E	2.29
Hermanus	26	34.42S	19.22E	4.90
Pic-du-Midi	2860	42.93N	0.25E	5.36

(Super NM-64) extending over the period 1965–1975. The altitude, geographic coordinates and cut-off rigidity of each station are listed in Table I. The data (corrected for pressure) for each station were normalized by

$$\frac{I_i - I_{\min}}{I_{\max} - I_{\min}},$$

where $I_{\min}$ and $I_{\max}$ are, respectively, the minimum and maximum intensities of cosmic rays during the 20th solar cycle and I_i is the corresponding half-year value of cosmic-ray intensity. With this method the intensities at solar minimum (1965) are taken equal to 1.00 and at solar maximum (1969) are taken equal to zero.

For this analysis we have also used the semi-annual number of significant solar proton events N_p (Sharley and Kroehl, 1977; Sharley *et al.*, 1979), and the half-yearly averages of relative sunspot number R (Zürich Observatory) and geomagnetic index A_p.

A detailed study of these data led to a new generalized empirical relation. Accordingly the cosmic-ray intensity which is observed in the Earth (modulated intensity) on a semi-annual basis can be calculated from the difference between constant function C and the sum of the most important solar and terrestrial indices which are affected cosmic-ray modulation. This expression, taking into account the indices R, N_p and A_p, is of the form

$$I = C - 10^{-3}(KR + 4N_p + 12A_p)\,, \tag{2}$$

where C is a constant which depends linearly on the cut-off rigidity of each station and K is a coefficient which is also rigidity-dependent and is probably related to the diffusion coefficient of cosmic rays and its transition in space. The physical properties in the modulating region derived from the constant C and the

TABLE II
Values of the constant C and the coefficient K for each of the neutron monitoring stations that have been used in this work

Station	C	K
Alert	$C = 0.93$	$K = 5.0$; 1965I–1967II.
		$K = 5.0 + 2.0 \sin \frac{1}{6}\pi t$; 1968I ($t = 0$)–1971II ($t = 7$)
		$K = 0.0$; 1972I–1975II
Thule	$C = 0.93$	$K = 4.0$; 1965I–1968I
		$K = 7.0–0.5 \sin \frac{1}{2}\pi t$; 1968II ($t = 0$)–1971I ($t = 5$)
		$K = 0.0$; 1971II–1975II
McMurdo	$C = 0.94$	$K = -2$; 1965I–1966I, 1971II–1975II
		$K = 4$; 1966II–1967II
		$K = 5 + 2 \sin^2 \frac{\pi}{6} t$; 1968I ($t = 0$)–1971I ($t = 6$)
Inuvik	$C = 0.94$	$K = 1.5$; 1965I–1966I, 1971II–1975II
		$K = 1.5 + 5.4 \sin \frac{\pi t}{12}$; 1966II ($t = 0$)–1971I ($t = 9$)
Goose Bay	$C = 0.95$	$K = 1.9$; 1965I–1966I, 1971II–1975II
		$K = 1.9; + 4.8 \sin \frac{\pi t}{12}$; 1966II ($t = 0$)–1971I ($t = 9$)
Deep River	$C = 1.02$	$K = 1.9$; 1965I–1966I, 1971II–1975II
		$K = 1.9 + 5.7 \sin \frac{1}{12}\pi$; 1966II ($t = 0$)–1971I ($t = 9$)
Kiel	$C = 1.09$	$K = 5.5$; 1965I–1968I, 1973II–1975II
		$K = 7.7 + 0.7 \cos \pi t$; 1968II ($t = 0$)–1971I ($t = 5$)
		$K = 5.5 - 3.5 \sin \frac{1}{4}\pi t$; 1971II ($t = 0$)–1973I ($t = 4$)
Hermanus	$C = 1.27$	$K = 10.0 - 2.5 \sin \frac{1}{6}\pi t$; 1965II ($t = 0$)–1968II ($t = 6$)
		$K = 10.0 - \quad \sin^2 \frac{1}{2}\pi t$; 1968II ($t = 0$)–1970II ($t = 4$)
		$K = 10.0 - 6 \sin \frac{1}{4}\pi t$; 1971I ($t = 0$)–1975I ($t = 8$)
		$K = 10.0$; 1965I, 1975II
Pic-du-Midi	$C = 1.30$	$K = 10.0 + 4.5 \sin \frac{1}{4}\pi t$; 1964II ($t = 0$)–1968II ($t = 8$)
		$K = 10.0$; 1969I–1970II
		$K = 10.0 - 4.5 \sin \frac{1}{4}\pi t$; 1971I ($t = 0$)–1975I ($t = 8$)

coefficient K are discussed below, while their numerical values are given for each station in Table II.

The observed neutron monitoring data of each station, I_{obs}, and the corresponding I_{cal} values calculated from Equation (2) are given in Table III. The 11-year variation of these values is shown in Figure 1. The continuous line represents the observed cosmic-ray intensity I_{obs} and the dashed line gives the value of I_{cal}. It is worth mentioning that for all nine neutron monitoring stations, the agreement between the measured cosmic-ray intensities and those calculated by Equation (2) is very good. The standard deviation between the observed and calculated values of cosmic-ray intensity is of the order of 5–9%.

If we subtract I_{cal} from I_{obs}, the difference $\Delta(I_{obs} - I_{cal})$ should be independent of the 11-year and short-term variations. Practically, however, the difference $\Delta(I_{obs} - I_{cal})$ in Figure 2 still shows remarkable short-term variations, especially

TABLE III

Percentage values of the intensity of cosmic-rays, measured, I_{obs}, and calculated by relation (2), I_{cal}

	Alert		Thule		McMurdo		Inuvik		Goose Bay		Deep River		Kiel		Hermanus		Pic du Midi	
	I_{obs} (%)	I_{cal} (%)	I_{obs} (%)	I_{cal} (%)	I_{obs} (%)	I_{cal} (%)	I_{obs} (%)	I_{cal} (%)	I_{obs} (%)	I_{cal} (%)	I_{obs} (%)	I_{cal} (%)	I_{obs} (%)	I_{cal} (%)	I_{obs} (%)	I_{cal} (%)	I_{obs} (%)	I_{cal} (%)
1965I			1.00	0.77	1.00	0.87	1.00	0.83	1.00	0.83	1.00	0.89	1.00	0.91	1.00	1.03	1.00	1.01
1965II	1.00	0.76	0.95	0.78	0.98	0.88	0.96	0.82	0.94	0.83	0.92	0.90	0.91	0.91	0.94	1.02	0.85	0.99
1966I	0.87	0.64	0.83	0.68	0.86	0.91	0.89	0.78	0.85	0.78	0.86	0.85	0.82	0.79	0.88	0.85	0.77	0.72
1966II	0.58	0.48	0.54	0.54	0.58	0.55	0.62	0.69	0.58	0.68	0.57	0.74	0.55	0.61	0.61	0.66	0.52	0.56
1967I	0.44	0.31	0.35	0.40	0.40	0.41	0.48	0.50	0.43	0.49	0.44	0.54	0.43	0.43	0.48	0.43	0.42	0.52
1967II	0.34	0.28	0.33	0.37	0.34	0.38	0.36	0.35	0.25	0.36	0.37	0.39	0.34	0.38	0.32	0.34	0.37	0.59
1968I	0.25	0.20	0.26	0.31	0.21	0.21	0.28	0.19	0.25	0.19	0.30	0.20	0.28	0.31	0.24	0.14	0.27	0.38
1968II	0.02	0.09	0.00	−0.01	−0.01	0.10	0.03	0.08	0.04	0.10	0.03	0.09	0.03	0.00	0.03	0.01	0.04	0.04
1969I	0.00	−0.05	0.00	−0.02	0.00	−0.04	0.00	−0.04	0.00	−0.01	0.00	−0.04	0.00	0.07	0.00	0.03	0.00	−0.06
1969II	0.02	0.08	0.03	0.08	0.06	0.09	0.03	0.10	0.02	0.13	0.03	0.11	0.04	0.10	0.11	0.12	0.10	0.15
1970I	0.04	0.00	0.03	−0.10	0.04	0.01	0.07	0.01	0.02	0.04	0.06	0.00	0.06	0.12	0.13	0.07	0.12	−0.01
1970II	0.02	0.19	0.09	0.09	0.10	0.20	0.08	0.18	0.06	0.20	0.08	0.20	0.08	0.12	0.15	0.15	0.16	0.18
1971I	0.28	0.49	0.39	0.40	0.50	0.50	0.42	0.48	0.34	0.49	0.46	0.53	0.45	0.54	0.49	0.55	0.51	0.58
1971II	0.61	0.48	0.77	0.79	0.82	0.95	0.66	0.68	0.60	0.66	0.77	0.73	0.70	0.71	0.72	0.68	0.71	0.63
1972I	0.77	0.67	0.76	0.67	0.82	0.83	0.68	0.57	0.52	0.55	0.79	0.62	0.73	0.68	0.82	0.71	0.66	0.63
1972II	0.71	0.75	0.74	0.75	0.81	0.88	0.67	0.67	0.57	0.65	0.75	0.72	0.68	0.72		0.74	0.64	0.70
1973I	0.65	0.68	0.66	0.68	0.71	0.78	0.59	0.62	0.49	0.61	0.70	0.69	0.60	0.59		0.57	0.54	0.60
1973II	0.78	0.77	0.80	0.77	0.92	0.84	0.78	0.74	0.68	0.73	0.89	0.80	0.76	0.77		0.68	0.72	0.75
1974I	0.75	0.71	0.71	0.71		0.78	0.72	0.68	0.60	0.67	0.79	0.74		0.69		0.54		0.62
1974II	0.61	0.68	0.67	0.68		0.76	0.57	0.63	0.48	0.62	0.64	0.70		0.64		0.50		0.57
1975I	0.76	0.74	0.73	0.74		0.77	0.72	0.73	0.64	0.74	0.85	0.81		0.84		0.98		1.01
1975II	0.80	0.78	0.77	0.78		0.83	0.77	0.76	0.65	0.76	0.89	0.83		0.83		0.92		0.95

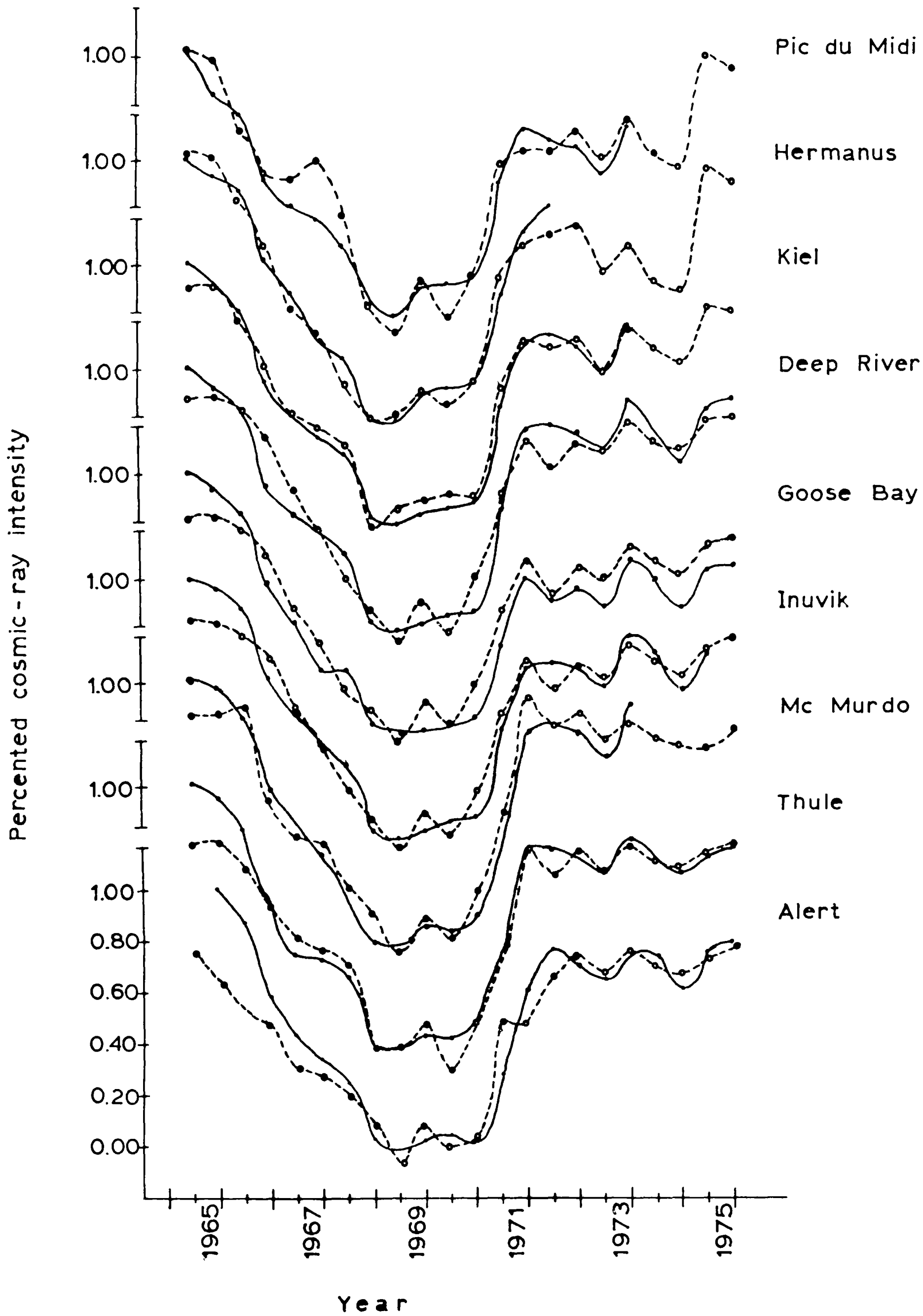

Fig. 1. The 11-year variation of cosmic-ray intensity for each station is given. The continuous line represents the observed cosmic-ray intensity I_{obs}, and the dashed line gives the corresponding value I_{cal}, calculated by relation (2).

during the years 1965–1966 due, perhaps, to the incomplete elimination by the present indices.

Examining the above relation (2) and applying this to the nine ground-based stations detecting cosmic rays, we observe that the constant C as the mean

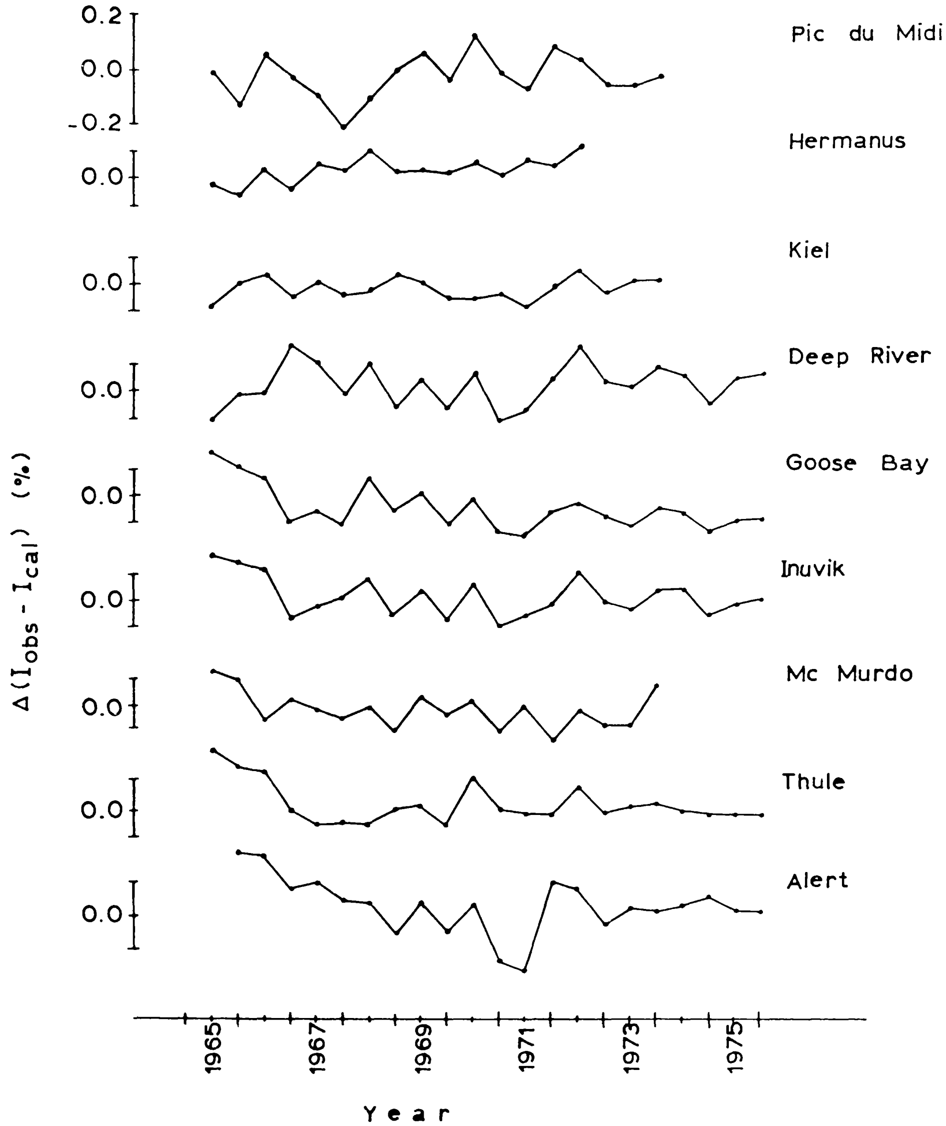

Fig. 2. Differences of the observed cosmic-ray intensities I_{obs} and the I_{cal} values calculated by relation (2) for each station from 1965–1975.

value of the coefficient $K(\bar{K})$ are linearly correlated with the cut-off rigidity of each station for the 20th solar cycle. The variation of C and $\bar{K}$ versus the rigidity of the stations are presented in Figure 3. A small discrepancy from the linear correlation was shown in the value of $\bar{K}$ from the McMurdo neutron monitor. From Figure 3 we derive the relation

$$C = 0.93 + 0.07P, \tag{3}$$

$$\bar{K} = 3.00 + 1.36P, \tag{4}$$

where P is the cut-off rigidity of each station. From an off-hand point of view, the coefficient K is a quantity of the modulation of cosmic rays travelling

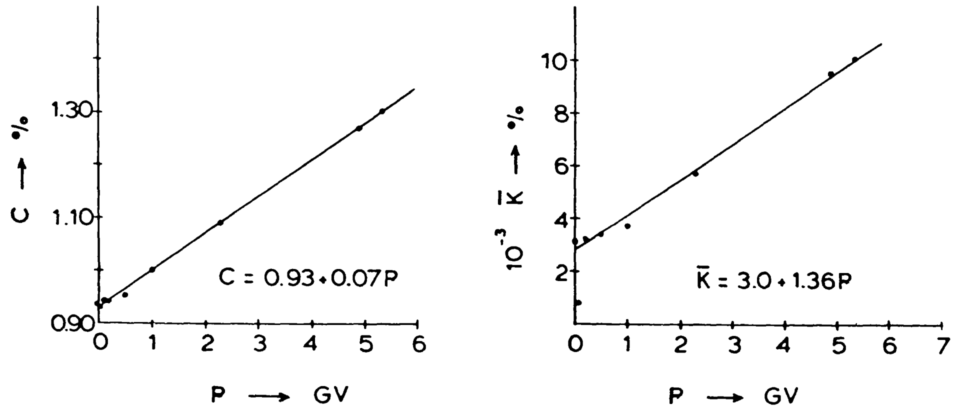

Fig. 3. Rigidity dependence of the constant C and coefficient K for the time interval 1965–1975.

through interplanetary space with the solar wind. The time dependence of semi-annual values of this coefficient for each station is given in Figure 4.

It is interesting to note that the coefficient K has a constant value for the first years of the ascending branch of solar activity and for the last 3–4 years of the descending branch, while for the maximum solar activity it has a period of 4–6 years and can be presented by the relation

$$K = a + b \sin \frac{2\pi}{\Omega} t, \tag{5}$$

where a and b are constants given in Table II for each station. It is noted that the stations with cut-off rigidities $\geqslant 2.20$ GV, in which there is a smaller modulation of cosmic rays, there appears to be a multiple of variations in the frequency of K, which probably results from slight variations in the wide asymptotic cones of acceptance of these stations, which have been introduced over a period of 2–3 years.

In the low-energy stations ($P \leqq 2.20$ GV) the curve of K is in inverse correlation with the curve which shows the size of the polar coronal holes; this is presented in Figure 4 (Hundhausen *et al.*, 1980). As was recently pointed out, there is a close correspondence between the size of the polar holes and the variations in cosmic-ray intensity. This suggests that a three-dimensional interplanetary structure influences the propagation of cosmic rays through the solar system to the Earth's orbit. It is known that coronal holes are associated with magnetic field lines which open into interplanetary space, and have been identified as the source of the major streams of fast solar wind in interplanetary space. Coronal holes also play a key role in determining the spatial structure of the interplanetary magnetic field. Thus, there is a good connection between the variation of the coefficient K and the size of coronal holes and, consequently, with the structure and variations of the interplanetary magnetic field.

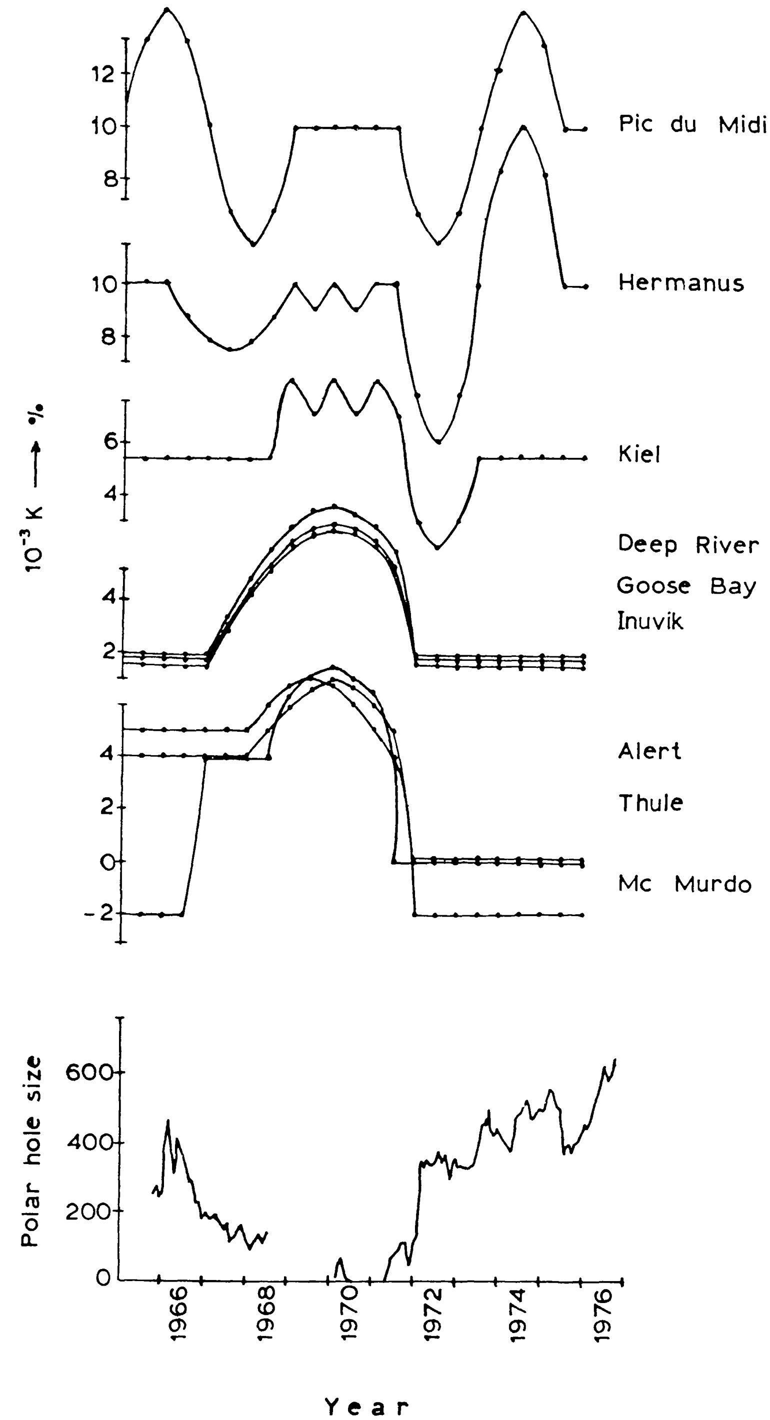

Fig. 4. Time dependence of semi-annual values of the coefficient K for each station. Also, the variation of polar hole size versus time is presented for the time interval 1965–1975.

3. Some Characteristics of the 11-Year Variation of Cosmic-Ray Intensity in the 20th Solar Cycle

A detailed examination of Figures 1 and 2 and of the 11-year variation of cosmic rays reveals some interesting features.

A graph of the variation of cosmic-ray intensity over the 20th solar cycle bears a close relationship to the actual solar activity cycle. The two maxima first postulated by Gnevyshev (1967) in a solar cycle also seem to be detectable in cosmic rays also during a solar cycle. Cosmic-ray intensity appeared at two minima: the first appeared in 1969, which coincided with maximum solar activity, and the second appeared at the end of 1971 (Křivský and Růžičková-Topolová, 1978). The sudden reappearance of the polar holes between late 1970 and early 1971, following their disappearance during sunspot maximum, has been justified by the second minimum of cosmic rays. It was also noted that the temporal variations in the size of the polar holes corresponded to the variations in cosmic-ray intensity observed at the Earth (Hundhausen *et al.*, 1980). Between the two minima of cosmic rays there occurred a polarity reversal of the magnetic field of the Sun because of the 22-year variation. The polarity reversal took place in the southern hemisphere in mid-1969 and ended in August 1971 when the northern hemisphere completed its reversal.

It is noteworthy that the amplitude of modulation in the solar cycle examined is smaller than that in the 19th cycle. Also the correlation between the cosmic-ray intensity variations and the solar activities is poor compared with the previous solar cycles (Ashirof *et al.*, 1977). Moreover, some anomalous phenomena in the modulation of cosmic rays were observed for several years after the solar maxima; e.g., the abnormality of the modulation rigidity spectra of cosmic-ray intensities (Lockwood and Webber, 1979), the sudden recovery of the intensity (Kuzmin *et al.*, 1977), etc. Recently, many researchers have pointed out that all these strange features at the 20th cycle could be explained by the superposition of 22-year and 11-year modulations. It is noted that the rigidity spectra of these two modulations are different from each other (Charakhchyan *et al.*, 1977) and the 22-year modulation is independent of solar activity, except for its transition period (Ashirof *et al.*, 1977).

It is mentioned that the time lag between cosmic rays and solar activity in the 20th solar cycle is not significant; it is only two months (Nagashima and Morishita, 1980b) – therefore the hysteresis effect appears to have been reduced considerably in this solar cycle.

4. Discussion and Theoretical Interpretation of Relation (2)

It is well known that the solar cycle modulation of the propagation of cosmic rays entering the solar system from interstellar space has been attributed to their interaction with a solar wind that varies with solar activity. A detailed theory of such effects of scattering of cosmic rays by irregularities in the magnetic field convected along by the solar wind has been developed (Parker, 1963; Jokipii and Parker, 1970). The diffusion–convection and adiabatic deceleration theory (Gleeson and Axford, 1967) of galactic cosmic rays into a spherically symmetric solar wind with this scattering would lead to an 11-year variation. In the light of this theory, the modulations are well explained by setting proper physical states in the modulating region, but it is not so clear how the states are related to solar activities.

Iucci *et al.* (1975) tried to obtain a dynamic relation of the modulation of cosmic rays to solar activity, assuming the following two mechanisms. One is an outward-sweep-away mechanism from the Sun due to the flare activity that causes a depression of the cosmic-ray density; the other is a diffusion mechanism which causes a recovery of the density.

Contrary to Iucci *et al.*, whose model treats the modulation as non-stationary, the coasting solar wind model (Simpson, 1963) interprets it as a variation in quasi-stationary state. With this concept it is assumed that disturbances due to solar activities continue to affect cosmic rays while travelling through the modulating region with the solar wind. In other words, the intensity of cosmic rays at a time t is affected by all the activities produced from the Sun before the specified time t. Accordingly, the modulation can be described by the following integral equation which is derived from a generalization of Simpson's coasting solar wind model (1963) in the form

$$I(t) = I_\infty - \int_0^\infty f(r) S(t-r)\,\mathrm{d}r, \tag{6}$$

where I_∞ and $I(t)$ are, respectively, the galactic and modulated cosmic-ray intensities, $S(t-r)$ is the source function representing some proper solar activity index at a time $t-r(r \geqslant 0)$ and $f(r)$ is the characteristic function which expresses the time dependence of an efficiency depression due to solar disturbances represented by $S(t-r)$, when the disturbances propagate through the modulating region with the solar wind. It is noteworthy, as Nagashima and Morishita (1980a) have pointed out, that this equation can also be derived from the spherically symmetric diffusion–convection theory, including the Compton–Getting factor (Gleeson and Axford, 1967) on some assumptions, and the source and characteristic function can acquire new physical meanings which are related to the diffusion coefficient of cosmic rays and its transition in space. Nagashima and Morishita (1980a) have shown that the modulations can be described by the source function which is expressed by the following linear combination of two indices: one is the sunspot number R and the other the geomagnetic activity index AA substituted for such stream-like disturbances as coronal holes (Murayama and Hakamada, 1975),

$$f(r) S(t-r) = f_R(r) R(t-r) + f_A(r) AA(t-r)\,. \tag{7}$$

In this work, the dependence of modulations and their surroundings on solar activity is studied by a new method, using data of cosmic-ray intensities from ground-based stations well distributed in latitude. According to this analysis it is proposed that the modulations ought to be expressed by the linear combination of three indices – the sunspot number R, the number N_p of proton events, and the geomagnetic activity A_p – i.e.,

$$f(r)S(t-r) = f_R(r)R(t-r) + f_N(r)N_p(t-r) + \\ + f_A(r)A_p(t-r)\,. \tag{8}$$

The time lag r between solar activity and cosmic-ray intensity in the examined solar cycle is approximately $\leqslant 2$ months (Nagashima and Morishita, 1980a). This time can be neglected in relation (8) because of the use of half-year values of all indices in the present analysis. Substituting Equation (8) into the general equation of Simpson's model and indentifying with the empirical relation (2) we get

$$I_\infty = C = 0.93 + U\,, \tag{9}$$

$$\int_0^\infty f_R(r)\,\mathrm{d}r = K \times 10^{-3}\,, \tag{10}$$

$$\int_0^\infty f_N(r)\,\mathrm{d}r = 4 \times 10^{-3}\,, \tag{11}$$

$$\int_0^\infty f_A(r)\,\mathrm{d}r = 12 \times 10^{-3}\,, \tag{12}$$

where U expresses the modulation of the galactic cosmic-ray intensity I_∞ due to the cut-off rigidity of each station. The characteristic function $f(r)$ of the indices N_p and A_p has a constant value during the 20th solar cycle, while the $f(r)$ distribution of the index R has a complex behaviour (Section 2).

This behaviour is explained by the fact that the existence of a 22-year variation affects the 11-year cosmic modulation, as is obvious from Figure 1. At the end of 1971, cosmic ray intensity made a sudden recovery to the pre-decrease level which occurs one year behind the polarity reversal of the polar magnetic field of the Sun. This time lag r can be explained on the basis of Simpson's model and the general diffusion–convection theory, and is expressed by two characteristic times as

$$r = r_{cs} + r_{DC}\,, \tag{13}$$

where r_{cs} is the time required for galactic cosmic rays to recognize the polarity reversal at the modulation boundary after the occurrence of the reversal at the solar surface, and r_{DC} is the time required for galactic cosmic rays to reach the Earth through the diffusion–convection process after receiving the information at the boundary (22 days for neutrons with $P = 1.5$ GV). If we accept relation (13) and the experimental Equation (4), the characteristic function of sunspot number R can be written as

$$\int_0^\infty f_R(r_{cs})\,\mathrm{d}r_{cs} + \int_0^\infty f_R(r_{DC})\,\mathrm{d}r_{DC} = (3.0 + 1.36P)F(t) \times 10^{-3} \tag{14}$$

where $F(t)$ is a function of time.

From this analytical expression we find that

$$\int_0^\infty f_R(r_{cs})\,\mathrm{d}r_{cs} = 3 \times 10^{-3} F(t)\,, \tag{15}$$

$$\int_0^\infty f_R(r_{DC})\,\mathrm{d}r_{DC} = 1.36P \times 10^{-3} F(t)\,. \tag{16}$$

Note that the characteristic function of sunspot number R reported to the characteristic time r_{cs} is independent of terrestrial parameters by definition of the time r_{cs}. Thus, the function $F(t)$ is not related to the rigidity of ground measured particles and other terrestrial indices. On the other hand, the characteristic function of R reported to the time r_{DC} *is* dependent on the rigidity P of the particles and the function $F(t)$, which can be related to solar and interplanetary parameters. Because of the definition of the time r_{DC}, the function $F(t)$ can be well related to the diffusion process of cosmic rays and its transition in interplanetary space.

As Nagashima and Morishita (1980a) have shown, the function $f(r)$ is inversely proportional to the transition of the diffusion coefficient due to the magnetic disturbances carried on the solar wind. It is known that the diffusion coefficient is related to magnetic fluctuations $\overline{\Delta H}$ in the modulating region. Their mutual relation is not so simple (Jokipii, 1967; 1968); however, if we assume that the diffusion coefficient is inversely proportional to $\overline{\Delta H}$, we obtain the fluctuations conversely from the observed coefficient. Consequently, $\overline{\Delta H}$ is assumed to be proportional to the function $f(r)$ and also to the coefficient K which is given by relation (2). Indeed, it was experimentally confirmed in the present work (Figure 3) that the coefficient K is in inverse relation with the size of the polar coronal holes. This correlation was poor for stations with cut-off rigidities >2.20 GV (Hundhausen *et al.*, 1980). As has been shown by King (1976), the yearly averaged magnitudes of positive and negative polarity magnetic field vectors portray separate solar cycle variations which are in inverse correlation with the variation in the sizes of the polar coronal holes. From the above it is evident that the characteristic function $f_R(r)$ and, consequently, the coefficient K gives information on the diffusion coefficient of cosmic rays.

5. Summary and Conclusions

From the above analysis and discussion we conclude the following:

The existence of an 11-year modulation of cosmic-ray intensity in the 20th solar cycle is pointed out, using the data from nine world-wide neutron monitoring stations over the period 1965–1975. Some anomalous phenomena appear in this solar cycle, such as the poor correlation between cosmic-ray intensity and solar activity, the sudden recovery of intensity, the small time lag between cosmic-ray intensity and solar activity, etc. These phenomena are associated with polarity reversal of the polar magnetic field of the Sun which occurs around the solar maximum. Thus, the modulation of cosmic-ray intensity is the result of the superposition of 22-year and 11-year modulations.

A fundamental equation which describes the long-term modulation of cosmic-ray intensity is given in this work. According to this relation the modulated cosmic-ray intensity that was measured by the ground based stations is equal to the galactic cosmic-ray intensity (unmodulated) at a finite distance – corrected by a few appropriate solar and terrestrial activity indices – which causes the disturbances in interplanetary space. Using the sunspot number R, the geomagnetic index A_p and the number of proton events N_p, the corresponding cosmic-ray intensities have been calculated by proper values of the constant C and coefficient K. The constant C has a constant value for each station, which is rigidity dependent, and the coefficient K is mainly responsible for the 11-year modulation of cosmic rays.

For low rigidities ($P \leqslant 2.20$ GV) this coefficient can be inversely correlated to the size of the polar coronal holes.

The above-mentioned relation is adequately explained by the generalized Simpson's solar wind model, where the constant C has physical meaning and the coefficient K is related to the diffusion coefficient of cosmic rays and its transition in space.

In conclusion, it is noteworthy that the analytical method which utilizes the empirical relation (2) is useful for the study of the long-term modulation of cosmic rays. Owing to the method used, we could reproduce to a certain degree the modulation with the proper source function (R, N_p, A_p) and could also associate the source function with the electromagnetic properties in the modulating region (K). Therefore, it is necessary to search for a more suitable source function among various kinds of solar activity indices or physical quantities.

In the future a further study of these parameters with a variety of phases or time lags, perhaps with observations out of the ecliptic plane, will lead us to a better understanding of the relations among coronal structure, interplanetary structure and cosmic rays in the solar system.

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