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Peak-Size Distributions of Proton Fluxes and Associated Soft X-Ray Flares

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Abstract A database combining information about solar proton enhancements (SPEs) near the Earth and soft X-ray flares (GOES measurements) has been used for the study of different correlations through the period from 1975 to May 2006. The emphasis of this work is on the treatment of peak-size distributions of SXR flares and SPEs. The frequency of SXR flares and solar proton events (>10 and >100 MeV, respectively) for the past three solar cycles has been found to follow mainly a power-law distribution over three to five orders of magnitude of fluxes, which is physically correct beyond the "sensitivity" problem with the smallest peak values. The absence of significant spectral steepening in the domain of the highest peak values demonstrates that during the period considered, lasting 30 years, the limit of the highest flare's energy release has not yet been achieved. The power-law exponents were found to be -2.19 ± 0.04, -1.34 ± 0.02, and -1.46 ± 0.04, for the total SXR flare distribution and the total SPE distributions (for both $E_{\mathrm{P}} > 10$ MeV and $E_{\mathrm{P}} > 100$ MeV), respectively. For SPEs associated with flares located to the West of 20° W, the exponents are -1.22 ± 0.05 ($E_{\mathrm{P}} > 10$ MeV) and -1.26 ± 0.03 ($E_{\mathrm{P}} > 100$ MeV). The size distribution for corresponding flares follows a power law with a slope of -1.29 ± 0.12. Thus, X-ray and proton fluxes produced in the same solar events have very similar distribution shapes. Moreover, the derived slopes are not incompatible with a linear dependence between X-ray flare power and proton fluxes near the Earth. A similar statistical relation is obtained independently from the direct comparison of the X-ray and proton fluxes. These all argue for a statistically significant relationship between X-ray and proton emissions.

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1. Introduction

Solar flares associated with proton enhancements are known as "proton" flares. From an astrophysical as well as a practical point of view these flares are of importance because of their potential hazard to spaceflight from the interactions of the energetic particles with matter and particularly human bodies. Hudson (1978) stated that the joint study of size distributions for neutral emissions and proton flux characteristics is a key to several important questions: *i*) Do proton flares differ fundamentally from ordinary flares or not? *ii*) Do proton flares represent the high, most powerful end of the flare distribution? *iii*) Does an energy threshold exist for proton generation in flares? These questions, formulated 29 years ago, are still unanswered today. But we have much more statistical information and a better possibility of tackling this problem.

The peak-size distributions of the electron and proton fluxes and other flare emissions have been studied by many researchers using different data sets with the aim of defining and comparing their shapes. The supplement of new information allows us to define more accurately the shape of peak-size distribution in the domain of the small peak values (nearly turnover position) and to extend this distribution for the events with greatest fluxes.

The similarity of distribution functions and multiple correlation analysis showed that the energies emitted in the optical, soft and hard X-ray (SXR and HXR, respectively), and γ-ray wavelengths as well as the energy carried away from the Sun by energetic particles (protons and electrons) can be useful equally as a proportional measure of the total flare energy (see, for example, Hudson, 1978; Kurt, Kuroscka, and Zenchenko, 1989; Kurt, 1990; Crosby, Aschwanden, and Dennis, 1993; Lee, Petrosian, and McTiernan, 1995; Bromund, McTiernan, and Kane, 1995; Feldman, Doschek, and Klimchuk, 1997). Aschwanden, Dennis, and Benz (1998) have published a quite complete table with the HXR and SXR distribution functions.

A summary of the slopes of power-law presentation of the distribution functions of the interplanetary energetic protons is given in Table 1. These results derived from earlier data sets indicate that the value of the power-law slope of the proton peak-size distributions is equal to $(-1.15)-(-1.40)$ (Van Hollebeke, Ma Sung, and McDonald, 1975; Belovsky and Ochelkov, 1979; Kurt, 1990; Cliver *et al.*, 1991). Subsequently the slope (ν) of peak/fluence proton size distributions was found in the range from -1.3 to -1.4 (Smart and Shea, 1997; Mendoza *et al.*, 1997; Belov *et al.*, 2005).

The present investigation follows the work of Hudson (1978) and Nonnsat and Armstrong (1982), who found the correlation between SXR flares and proton events. Their results also showed that the SXR flare energy can be used for estimating the probability and the intensity of the proton event. Today, ground-level and space-based experiments allow collection of a great deal of homogeneous data of different neutral solar flare emission as well as the performance of a more thorough statistical analysis of diverse data sets.

This work continues our long-term efforts to define statistical properties of SPEs as well as to obtain reliable indices for SPE forecasting (Kurt, 1990; Gerontidou *et al.*, 2001, 2002, 2006; Kurt, Mavromichalaki, and Gerontidou, 2002; Kurt *et al.*, 2004). This contribution is also an extension of our preceding study of the proton enhancements and their relation to the SXR flares, which was based on the measurements of SXR flares and SPEs over the period 1976 – 2003 (Belov *et al.*, 2005). The selection of proton events is described in detail in that paper. Our database is now updated to May 2006. Here we note only that the proton flux maximum was adopted as the main criterion of the proton enhancements.

A brief description of a relationship between the solar activity level and the SPE occurrence rate is presented in Section 3. The emphasis of this work is on the treatment of the

Table 1 Summary of size distribution of SPEs.

	Time period	Energy (MeV)	Threshold intensity (pfu)	Number of events	Power-law index	References
1	1967 – 1972	20 – 80	$> 10^{-4}$	163	1.15 ± 0.05	Van Hollebeke, Ma Sung, and McDonald (1975)
2		>10			1.40 ± 0.15	Belovsky and Ochelkov (1979)
2	1977 – 1983	24 – 34	$>10^{-3}$	92	1.13 ± 0.04	Cliver *et al.* (1991)
3	1981 – 1982	>25	$> 7 \times 10^{-4}$	36	1.45 ± 0.05	Kurt (1990)
4	1965 – 1996	>10	>10	170	1.47	Smart and Shea (1997)
5	1965 – 1996	>10	$> 10^{3}$	26	2.42	Smart and Shea (1997)
6	1955 – 1993	>10	>1	305	1.27 ± 0.02	Mendoza *et al.* (1997)
7	1955 – 1993	>10	>1	320	1.37 ± 0.05	Miroschnichenko (2001)
8	1955 – 1996	>10	>10	233	1.43 ± 0.03	Miroschnichenko (2001)
9	1955 – 1996	>10	>750	32	2.12 ± 0.03	Miroschnichenko (2001)
10	1976 – 1999	>10	>10	147	1.30 ± 0.02	Gerontidou *et al.* (2002)
11	1970 – 2002	>10	>10	253	1.36 ± 0.04	Kurt *et al.* (2004)
12	1975 – 2003	>10	>0.1	1144	1.37 ± 0.03	Belov *et al.* (2005)
13	1975 – 2003	>100	>0.01	547	1.47 ± 0.03	Belov *et al.* (2005)
14	1975 – 2006	>10	>0.3	1265	1.34 ± 0.02	This work
15	1975 – 2006	>100	>0.1	639	1.46 ± 0.03	This work
16	1975 – 2006	>10	>0.3	253	1.22 ± 0.05	This work (20° W – 75° W)
17	1975 – 2006	>100	>0.1	142	1.26 ± 0.03	This work (20° W – 75° W)

peak-size distributions of SXR flares (Section 4) and those of SPEs (Section 5). The size distributions of all SPEs and SPEs, for which the association with solar flares were established in our database, have been examined. The principal goal of this work is the improvement of the already existing methods of solar proton event forecasting.

2. Data Selection and Method of Analysis

More than 62 000 solar flares were identified by GOES SXR monitoring in the 1 – 8 Å range from September 1975 through May 2006. The main parameters of all these flares have been entered into our database. Among them are the values of the X-ray maximum flux and estimations of the fluences. We separated 1,265 SPEs with proton energy $E_P >$ 10 MeV, 673 of which succeeded to be reliably associated with flares (Kurt *et al.*, 2004; Belov *et al.*, 2005). In comparison with Belov *et al.* (2005) we added to that database almost 5,000 SXR flares and 121 new selected proton events. The majority of these events occurred during recent years, but we made some inclusions and corrections for the older years also.

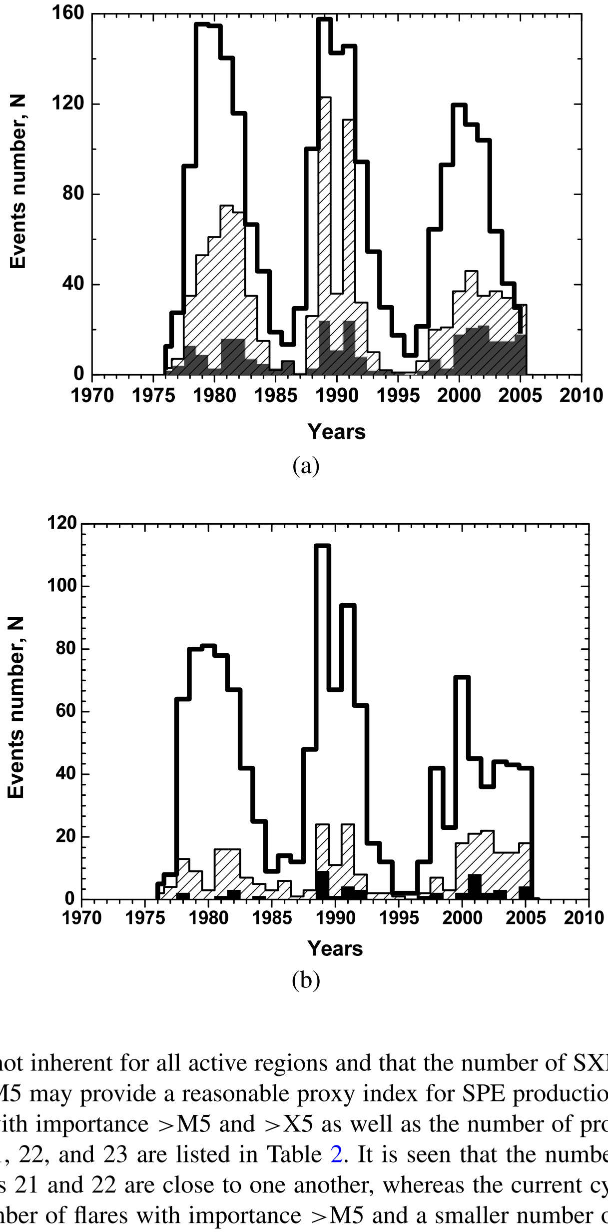
The selection of SXR flares and SPEs near the detector sensitivity threshold is often troublesome. The threshold values of the X-ray and particle fluxes depend on a constant factor of detector sensitivity and transitory factors determined by solar activity. Quiet periods made it possible to identify 10-MeV proton enhancements with 0.01 – 0.02 pfu over the background (*i.e.*, two to three orders below the threshold criterion for NOAA designated SPEs). Problematic cases usually pertain to small enhancements. However, the probability of energetic flares depends strongly on the magnetic configuration of the active region (Zirin, Sammis, and Tang, 2000; Jing *et al.*, 2006). As a result, major flares and proton events are observed rather often in series when a single active region generates several proton events one after another. During such periods of high levels of flare activity, when the proton enhancements follow one after another, the selection of separate proton events becomes much more complicated, often resulting in failure, because it is difficult to discriminate even the large SPEs in the decay phase of a previous very large proton event.

3. Temporal Distribution of Proton Events

The Wolf number (R_z) is often considered as the main and unique index of solar activity. However, a comparison of the yearly averaged R_z values over the past three cycles with the flare rates demonstrates that in general flare rates do not follow the behavior of the sunspot number (see also Bai, 2006). The time dependence of the yearly R_z values and the yearly rates of the SXR flares with importance >M5 are presented in Figure 1a. A very good similarity between the temporal distribution of ≥M5 SXR flares and SPE number is evident in this figure. Yearly values of the total number of SPEs with $E_P >$ 10 MeV derived from our data set, together with a rate of SPEs with $E_P >$ 10 MeV and flux $I_P >$ 10 pfu (by NOAA classification) and the number of the most intense proton events with $E_P >$ 10 MeV and $I_P >$ 500 pfu as well, are plotted in Figure 1b. On the basis of yearly numbers for a period of 31 years in total a high correlation ($r = 0.90$) is obtained between the number of SPEs with $E_P >$ 10 MeV and $I_P >$ 10 pfu and the number of the major flares (>M5).

Analysis of the monthly numbers in different subsets of SXR flares with importance from ≥M1 to ≥X3 and SPEs gives a maximum correlation coefficient $r = 0.74$ for the flares of importance ≥M5 (Belov *et al.*, 2005). The correlation coefficient between the monthly averaged SPEs and sunspot numbers is noticeably smaller ($r = 0.65$). This result indicates that

Figure 1 (a) Yearly values of the Wolf number R_z (thick line), the number of SXR flares with importance >M5 (hatched area), and the number of SPEs with $E_P > 10$ MeV and $I_P > 10$ pfu (black area). (b) Yearly values of the number of all SPEs with $E_P > 10$ MeV (thick line), the number of SPEs with $E_P > 10$ MeV and $I_P > 10$ pfu (thatched area) and the numbers of SPEs with energy >10 MeV and $I_P > 500$ pfu (black area).



energetic flare production is not inherent for all active regions and that the number of SXR flares of importance >M4 – M5 may provide a reasonable proxy index for SPE production rates. The number of flares with importance >M5 and >X5 as well as the number of proton events for solar cycles 21, 22, and 23 are listed in Table 2. It is seen that the number of these parameters for cycles 21 and 22 are close to one another, whereas the current cycle 23 presents a smaller number of flares with importance >M5 and a smaller number of total SPEs also. It is noteworthy that the number of the great SPEs (>10 MeV, >10 pfu) increased in the current cycle. The number of flare-associated SPEs seems to be almost the same during the three solar cycles.

4. Frequency Distribution of SXR Peak Fluxes

In this work, the peak fluxes of two different subsets of the SXR flares were analyzed: all SXR flares and flares associated with SPEs.

Table 2 Number of events in solar cycles 21, 22, and 23.

Solar cycle	21	22	23
SXR ≥ M5	361	343	269
SXR ≥ X5	18	22	16
Total >10 MeV SPEs	469	429	367
SPEs >10 MeV, >10 pfu	80	75	119
SPEs > 100 MeV, >0.01 pfu	216	222	201
Flare-associated SPEs	226	236	211

Recall that a distribution function differentiated with respect to event values (spectrum) $\Psi(\Phi)$ is defined as

$$\Psi(\Phi) = \mathrm{d}N(\Phi)/\mathrm{d}\Phi, \quad (1)$$

where N is the total number of events, $\mathrm{d}N$ is the number of events with Φ in the interval $\mathrm{d}\Phi$, and Φ is any characteristic property of neutral emission of solar flare or charged-particle fluxes (the protons, ions, or electrons of solar energetic particles).

The distribution function exhibits the following characteristics:

- A turnover or flattening in the smallest Φ values that are accessible for the measurements under review.
- For the event's type j above the turnover, it can be represented by a power law over several orders of magnitude according to the relation

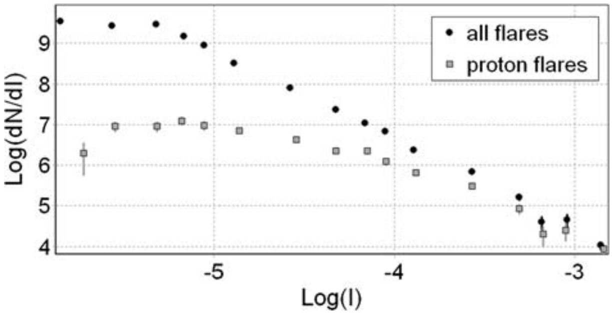
  $$\Psi_j(\Phi) = C(\Phi)^{-\nu}, \quad (2)$$

  where C and ν are constants determined from a least-square fit to the data.
- Plausible softening of the spectrum or break point for the greatest values of Φ.

The missed events are the cause of the turnover of the distribution function near the lesser values of Φ. The background problem is important for identifying the flares and proton events. Therefore each experimental distribution function carries reliable information on the flare activity only beyond the small intensity turnover. It may be that the softening of the right part of the distributions has a physical origin. Since the flare energy release has an upper limit, defined by the size of flare or acceleration region and the value of magnetic energy, the fluxes and fluences of any emission have to be limited. Consequently, we may expect an increase of the slope near the higher flux and higher fluence limits. The question is whether our measurements have progressed to this point. Strictly speaking, we are watching the flare activity for only a short time so that we are only approaching the limiting values of flare fluxes and fluences from the left. As a rule the events with the greatest values have very poor statistics in the available data sets. Consequently, when statistics are augmented by extending the time span the spectrum softening appears to be shifted to higher values. For example, this happened when SMM HXR events were growing in comparison with previous studies (Crosby, Aschwanden, and Dennis, 1993).

The $\log(\mathrm{d}N/\mathrm{d}I)$ *versus* $\log(I)$ distribution of all SXR flares compared with the corresponding one for the SXR flares associated with SPEs are depicted in Figure 2. The large number of events (62 109 SXR flares) allows us to find the linear portion of the distribution and the power-law slope (ν_{X}) with great confidence. Often the SXR background is higher than C1 importance (Feldman, Doschek, and Klimchuk, 1997) and sometimes it is higher than M1. This means that there are many missed flares in the range B1 – M1 caused by the

Figure 2 Peak-size distributions of the total number of SXR flares and of the proton-associated flares.



appearance of turnover in this flare importance interval. Thus, we are forced to remove all data points up to C4 importance, retaining only 16,697 flares. The resulting slope within a range of $4 \times 10^{-6} - 2 \times 10^{-3}$ W m^{-2} (from C4 to X3 importance) is $\nu_X = 2.19 \pm 0.04$.

Our slope (ν_X) is higher than those in previous publications. Note that the soft X-ray distribution has been studied more rarely than the HXR peak-size distribution. The first and most often cited SXR result is the study by Drake (1971), who obtained $\nu_X = 1.75 \pm 0.10$ from *Explorer* 33 and *Explorer* 35 measurements in the wavelength range 2 – 12 Å during 1965 – 1968. This value applies to flares with power in the range $10^{-6} - 10^{-4}$ W m^{-2} (C1 – X1). Drake did not have a sufficient number of powerful flares (≥X1) to carry out a statistical analysis. From the other side, a limited number of events (3,154) forced him to use relatively weak flares including those of C1 importance. To reduce the effect of variable background he applied a special method of correction. However, the background problem was not entirely resolved, and Drake demonstrates a strong dependence of his results on the low threshold. When a threshold of C1 was used, he got $\nu_X = 1.66$, and below a threshold of C2 he got $\nu_X = 1.84$. It is important to mention that Drake studied not the maximum X-ray power (as in the present work) but the power after background subtraction.

The value $\nu_X = 1.88 \pm 0.21$ obtained by Feldman, Doschek, and Klimchuk (1997) was based on the GOES (1 – 8 Å) data during the short period near minimum of solar activity (November 1993 – July 1995) when there was no powerful flare activity.

Lee, Petrosian, and McTiernan (1995) obtained $\nu_X = 1.86 \pm 0.10$ from GOES measurements (1 – 8 Å), carried out from 1980 to 1989. However, Lee, Petrosian, and McTiernan (1995) studied the peak-size distribution only for flares where both SXR and HXR were observed – it was a partial distribution. In the same work, the slope $\nu_X = 1.71 \pm 0.01$ was obtained for the Ca XIX line in the measurements onboard the SMM. Unfortunately, this value was given without any explanation so does not allow us to judge the peculiarities of this distribution. Recently, Yin *et al.* (2002) reported the slope $\nu_X = 2.14$ for the soft X-ray flares recorded by GOES satellites during solar cycle 22.

One can see that the majority of the aforementioned earlier results could not be directly compared with the results of the present work. Some of them have been inferred by different measurements and approaches to event selection. Data from GOES, if used, have been processed for separate especially selected periods. The direct comparison is possible only with the Yin *et al.* (2002) paper, where the obtained slope is $\nu_X = 2.14$, which is very close to our result. While this paper was being prepared for publication Hudson (2007) reported a slope of 2.19 within the $10^{-5} - 10^{-3}$ range of SXR flares.

It is unlikely to be surprised by the difference between the size distributions of SXR and HXR. The last one is flatter, with $\nu_X \cong 1.7$ (*e.g.*, Crosby, Aschwanden, and Dennis, 1993). In contrast, the size distributions of UV in nanoflares and microflares give slopes between 2.0 and 2.6 (Krucker and Benz, 1998; Parnell and Jupp, 2000).

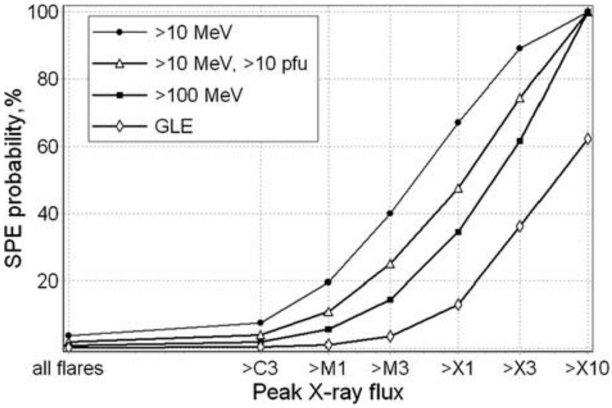


Figure 3 Dependence of the SPE probability on the X-ray flares importance for 20° E – 90° W heliographic longitude range. The proton events with $E_P > 10$ MeV are depicted by filled circles and events with proton flux >10 pfu by filled rectangles; >100 MeV proton events are presented by open triangles and GLEs by diamonds.

We deliberately do not depict in Figure 2 the power-law fits to the SXR flares associated with SPEs because of the significant deviation of this size distribution from the expected power law. One would expect a relationship between the distribution functions constructed for various characteristics of events of a single type (Daibog *et al.*, 1989; Lee, Petrosian, and McTiernan, 1995). However, the distribution of SPE-associated flares may not obey the same shaped distribution obtained for all SXR flares.

It should be emphasized that the probability of an arbitrary flare being included in the SPE-associated subset depends essentially on the flare importance. The size distributions presented in Figure 2 are important for evaluation of the probability of >10 MeV proton registration after SXR flares of specific importance. It should be taken into account that this probability depends strongly on the longitude of the associated flare as well.

The SPE probability for SXR flares of different importance is calculated throughout 1976 – 2006 and presented in Figure 3. To minimize the longitude effect, only western and central flares (*i.e.*, located to the West of 20° E) have been used. It is evident from Figure 3 that the SPE probability first becomes noticeable for the flare importance range C3 – M1. One out of four SXR flares of importance ≥M3 results in a measurable level of >10 MeV protons. At importance ≥X1, more than half of the flares associated with >10 MeV protons and ground-level enhancements (GLEs) become significant. Proton probability increases sharply above this level. Only 3 of 29 SXR flares of importance >X5 were not followed by SPEs. In this sample, a flare of the highest intensity (X6.2) not associated with protons occurred on 13 December 2001 in the eastern hemisphere (E09).

5. Peak-Size Distribution of Proton Events

During the period 1975 until the end of 2005, 1 265 events with $E_P > 10$ MeV and 646 events with $E_P > 100$ MeV were identified (but in the first half of 2006 not a single event was recorded). The peak-size distributions (dN/dI) of these two data sets are presented in Figure 4 for all events without regard to flare association and heliolongitude of the solar source. To avoid the wrong slope of the spectra caused by the turnover, the points with the smallest values (within the turnover region) have been removed before processing. In our case we removed 329 points of 1 265 (26%) for >10 MeV and 224 points of 639 (36%) for SPEs of >100 MeV. The power-law fit over the range 10^{-1} – 10^{4} pfu found by least squares is shown in this figure. The statistical weights (number of events) of the points were also

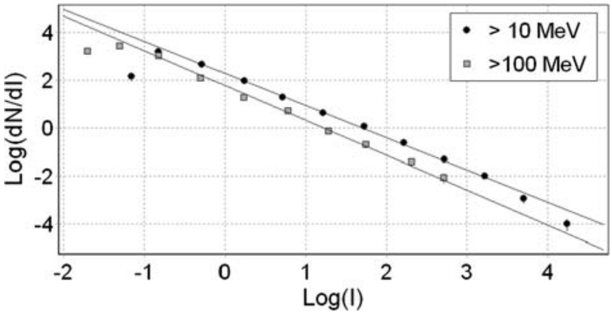


Figure 4 Peak-size distribution of SPEs with energy >10 MeV and with energy $E_P > 100$ MeV (all events independently from flare association). Lines represent the power-law fitting of the weighted data.

taken into account. The fitting parameters are summarized in Table 2. The previous results for the time interval 1976 – 2003 (Gerontidou *et al.*, 2002; Belov *et al.*, 2005) are shown for a comparison in Table 1. Additional data from the recent years improved our statistics and the new results may be considered as a refinement of the previous ones. The obtained exponent, 1.46 ± 0.04, for the distribution of >100 MeV events is slightly steeper than that for the events with $E_P > 10$ MeV (1.34 ± 0.02), but the difference is minor. It is not impossible that both distributions obey the same law.

Our linear approximations are valid within the intervals 0.3 – 3 000 pfu (>10 MeV) and 0.1 – 300 pfu (>100 MeV). It is possible that they will be valid also for the bigger proton fluxes, although we did not measure the highest fluxes the Sun can generate. It is obvious that our patterns will be changed in the future. During the very short time interval on the solar time scale (≈ 30 years) that was included in our analysis, none of the most powerful proton events occurred. Such events occurred in the past and they must be expected in the future. McCracken *et al.* (2001), analyzing the SPEs during a much longer time period (1561 – 1994), have concluded that the distribution of fluences of protons with $E > 30$ MeV became steeper for the higher fluxes.

In the case where proton productivity depends on the flare power, this dependence has to be interrelated with the shape of X-ray and proton size distributions. If $I_P \propto I_X^{\gamma}$, then the exponent γ is related to the slopes of associated SXR flare distribution (ν_X) and of the SPE distribution ν_P in the following way (Hudson, 1978; Crosby, Aschwanden, and Dennis, 1993):

$$\gamma = (\nu_X - 1)/(\nu_P - 1). \tag{3}$$

Hudson (1978) employed earlier results from Drake (1971) and Van Hollebeke, Ma Sung, and McDonald (1975) in Equation (3) and found γ to be $5.6^{+2.8}_{-1.4}$. The values ν_X and ν_P presented in this work lead to $\gamma = 3.5 \pm 0.1$ for $E_P > 10$ MeV and $\gamma = 2.6 \pm 0.1$ for $E_P > 100$ MeV. We see that the obtained $I_P(I_X)$ dependence is not as strong as that obtained from the previous, limited data. Nevertheless, these exponents are substantially bigger than one and confirm Hudson's conclusion about the threshold (overlinear) character of $I_P(I_X)$. We can also see some evidence of the threshold dependence in other results, for example in Figure 4. The most obvious threshold (near SXR flare importance C5 – M1) is seen in the dependence of the mean proton maximum flux on the X-ray flare peak flux, calculated by Belov *et al.* (2005) for all GOES X-ray flares up to 2003.

Additionally, to confirm this conclusion we have calculated the mean X-ray power of 673 flares well associated with SPEs and for 60 799 flares not associated with SPEs. The calculated mean values are $\langle I_X \rangle = (1.3 \pm 0.1) \times 10^{-4}$ W m^{-2} and $\langle I_X \rangle = (4.6 \pm 0.1) \times 10^{-6}$ W m^{-2}, respectively. The mean proton fluxes for the 673 proton flares are 250 ± 60 pfu (>10 MeV) and 9 ± 2 pfu (>100 MeV). One can easily see that the ratio of X-ray mean

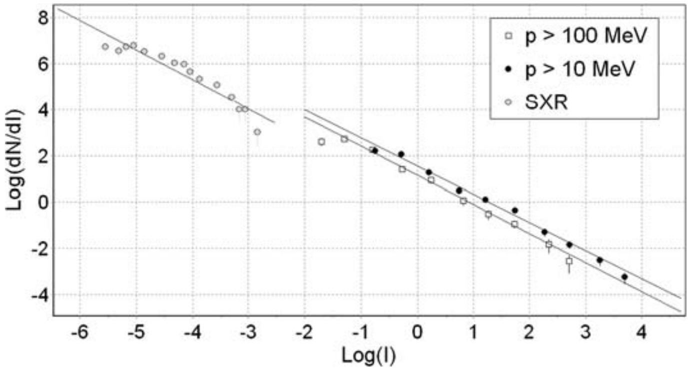


Figure 5 Peak-size distribution of all SPEs and SXR-associated solar flares on 20° W – 75° W longitudes. Lines represent the power-law fitting of the weighted data. X-ray flux I_X and the proton flux I_P are measured in W m^{-2} and pfu, respectively.

Table 3 Results of power-law fit of size distributions.

Subset	Power-law slope	Number of events	Number of events for fit
All X-ray flares	-2.19 ± 0.04	62 109	16 697
All SPEs with $E_P > 10$ MeV	-1.34 ± 0.02	1 265	936
All SPEs with $E_P > 100$ MeV	-1.46 ± 0.04	639	415
Proton flares in 20° W – 75° W	-1.29 ± 0.12	253	222
> 10 MeV SPEs with flares located in 20° W – 75° W	-1.22 ± 0.05	253	222
> 100 MeV SPEs with flares located in 20° W – 75° W	-1.26 ± 0.03	142	134

power for "proton" and "nonproton" flares is ≈ 28 and is substantially smaller than the ratio of mean proton fluxes to the corresponding proton background.

It is seen that not all X-ray flares are associated with proton events but only the most powerful of them. Therefore comparing the size distributions obtained for all X-ray flares and for the SPEs, we actually deal with substantially different sets of events, and therefore a significant distinction in the distribution shapes is understandable. To perform such a comparison more correctly we have to work with the SXR subset that is well associated with proton enhancements. Additionally, we ought to recall the strong dependence between proton fluxes measured near Earth and their solar source locations (for example, Belov *et al.*, 2005).

Bearing in mind the aforementioned considerations, we selected the proton events with well-associated flares within the heliographic longitude range 20° W – 75° W. We derived the size distribution slopes -1.22 ± 0.05 for the events with $E_P > 10$ MeV and -1.26 ± 0.03 for the events with $E_P > 100$ MeV (see Figure 5 and Table 3). Thus, for western sources, the slopes in different energy channels turned out to be closer to one another than those for all SPEs. These distributions are found to be flatter than the total SPE distributions. The reason may be that for western sources the association of relatively intensive SPEs can be defined with better certainty.

Furthermore, we derived the size distribution of X-ray fluxes for flares associated with proton events under consideration (Figure 5). Flares below the turnover of the proton fluxes were excluded. The best fit for SPE-associated flares within the 20° W – 75° W range gives a slope $\nu_X = -1.29 \pm 0.12$. This value is dramatically different from that found for all SXR flares ($\nu_X = 2.19$), but it is very close to the SPE slopes. The last result suggests that I_X and I_P in proton flares vary together. Then can use Equation (3) once again and recalculate the exponent γ with the new data. In this way we obtain $\gamma = 1.3^{+1.1}_{-0.7}$ for $E_P > 10$ MeV and $\gamma = 1.1^{+0.7}_{-0.4}$ for $E_P > 100$ MeV, which is consistent with a linear dependence between I_X and I_P.

Following Belov *et al.* (2005) and using the new expanded database we found the $I_P(I_X)$ dependence by more direct means of a log – log regression between I_P and I_X for the western events (20° W – 75° W):

$$I_P\ (> 10\ \text{MeV}) = (8.3 \pm 1.2) \times 10^4 I_X^{\gamma 1}, \tag{4a}$$

$$I_P\ (> 100\ \text{MeV}) = (9.7 \pm 1.6) \times 10^3 I_X^{\gamma 2}, \tag{4b}$$

where the X-ray flux I_X and the proton flux I_P are measured in W m^{-2} and pfu, correspondingly, $\gamma_1 = 0.93 \pm 0.10$ (correlation coefficient 0.54 for 209 events with energy >10 MeV and peak flux >0.3 pfu), and $\gamma_2 = 0.99 \pm 0.11$ (correlation coefficient 0.60 for 132 events with energy >100 MeV and peak flux >0.03 pfu). Equations (4a) and (4b) are the correction of similar equations of Belov *et al.* (2005), in which erroneous coefficients were given.

There is good agreement between relationships $I_P(I_X)$ derived from the size distributions and those obtained by the direct method. The slopes derived by power-law fitting are close to one another and are compatible with a linear $I_P(I_X)$ dependence. This result is not inconsistent with the threshold character of the $I_P(I_X)$ dependence, because associated flares represent the most powerful fraction of SXR flares and the majority of them are far above the threshold.

The I_P and I_X interrelation could be easily explained if we assumed that X-rays and accelerated protons are generated in a common process and energy release is distributed proportionally among various radiations. However, this simple explanation is not the sole explanation. If, as is often accepted (for example, Reames, 1995; Cliver, Kahler, and Reames, 2004), the acceleration is not related directly to a flare region, we should assume a more complicated scenario: a relation between CME energy and associated X-ray flare energy with some part of CME energy being transferred into particle acceleration, the “big flare syndrome” (Kahler, 1982).

We obtained two different estimations of power-law indices for $I_P(I_X)$. When we use a distribution of all SXR flares in the Hudson equation, we get $\gamma \approx -3$. If to use a distribution of only SPE-associated flares then the γ index reduces to ≈ -1. High values of the exponent index γ in the first case reflect mainly an increase of the proton-event probability with the growing power of the associated X-ray flare since $I_P = 0$ for almost all weak flares. In the second case, when we know that this is a proton event, we obtain from the Hudson equation an index (γ) that characterizes a real connection between X-ray flare power and proton flux intensity.

6. Conclusions

In this work a database of SXR flares and SPEs covering 31 years has been used. This database updated to May 2006 includes more than 62 000 SXR flares and 1 265 isolated proton enhancements with proton energy >10 MeV. Among them 673 events were well identified as flare related.

A high correlation exists between the number of SPEs and major (>M5) flares on a scale of months and years. The number of SXR flares with importance ≥M4 – M5 may provide a reasonable proxy index for SPE production rate.

This study supports the conclusion that "there is no repeatable pattern between the occurrence rates of individual events or episodes activity as a function of time within a specific solar cycle – albeit increasing phase, solar maximum or declining phase portion of the cycle" (Shea and Smart, 1995). The remarkable examples are the active regions that appeared in September 2005 and in December 2006 near the solar activity minimum, which produced strong flares and high-energy proton fluxes.

The peak-size distributions of various subsets of SXR flares as well as of the SPEs were derived with a better statistics than before.

It was found that fluxes over three to five orders of magnitude in each subset fit the power-law dependence with the following scaling exponents:

-2.19 ± 0.04 for the total SXR flares distribution.
-1.34 ± 0.02 for the total SPE distribution with $E_P > 10$ MeV.
-1.46 ± 0.04 for the total SPE distribution with $E_P > 100$ MeV.

We believe that power-law distributions are physically correct beyond the sensitivity and background problems for discrimination of both SXR flares and SPE events. Nevertheless, there are several indications that the powerlaw fits all the flares with very small energy release. For example, there are the hard X-ray "microflares" and small electron events that were observed by Lin *et al.* (1984) and Christe *et al.* (2004). Theoretically, a high-energy cutoff is required to keep the total flare energy limited. We did not find a deviation from the power-law behavior or a pronounced break point at the greatest peak values exceeding the statistical errors of the available data. The absence of a perceptible steepening suggests that during the observation period, covering 31 years, the upper limit of a flare's energy release has not yet been achieved. It is noted that SXR peak values for the partly occulted flares on 1 June 1991 (Kane *et al.*, 1995) and on 4 November 2003 (Kiplinger and Garcia, 2004) were estimated as importance X30 – X40.

The obtained results testify that the conditions of proton acceleration, escape from the acceleration regions, and propagation are arranged in such a way that the peak-size distributions of the SPEs result in the power-law dependencies. SPE distributions are based on measurement of the proton fluxes at 1 AU and *a priori* they will not be the same as those for the events on the Sun or at other distances from the Sun owing to both coronal and interplanetary propagation effects. The shape of size distributions for the flares of soft X-rays and for the solar protons reveal a big difference. The distribution of the proton fluxes is significantly harder than that for the X-ray flare power. Such a distinction is one of the consequences of the threshold character of a relation between X-ray flares and solar protons. Proton flares definitely represent the most powerful end of flare distribution.

The distinctions in the shape of X-ray flares and proton distributions should not be considered as evidence of the weak relationship between protons and flares. In fact, when we used only those flares associated with SPE flares and minimized the influence of the interplanetary propagation, the following slopes were obtained:

-1.29 ± 0.12 for the distribution of all SPE-associated flares located at 20° W – 75° W.
-1.22 ± 0.05 for the distribution of SPEs with $E_P > 10$ MeV associated with the flares located at 20° W – 75° W.
-1.26 ± 0.03 for the aforementioned SPEs with $E_P > 100$ MeV.

Thus, X-ray and proton fluxes produced in the same solar events have very similar distribution shapes. Moreover, the derived slopes point to the nearly linear relationship between X-ray flare intensity and proton fluxes near the Earth. The same statistical relation is obtained independently from the direct comparison of the X-ray and proton fluxes. Taken together, these results argue for a relation between X-ray and proton emissions. Let us notice that with this relation pure flare acceleration is not necessarily assumed and the relation is not inconsistent with possible acceleration by CME-driven shocks. X-ray and proton emissions seem to be related within complex solar events, including CME generation, in such a way that the energy release is distributed between X-ray radiation and accelerated particles.

In conclusion, we can say that the relationship between SXR flares and SPEs associated with them derived from the extended database will be helpful to solar – terrestrial physics studies and will offer a new means for the forecasting of SPE rates and proton fluxes as well.

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